Monetary Policy, Financial Constraints, and Redistribution[1]

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Abstract

This paper examines how financial constraints affect redistribution via monetary policy. We explore a novel mechanism of monetary non-neutrality, which is based on debt limits imposed in nominal terms. Specifically, when borrowing is constrained by current income, monetary policy can alter the real terms of borrowing. Changes in inflation exert ambiguous effects, depending on the initial debt/wealth position and the willingness to borrow. We show analytically that borrowers can benefit from increased debt limits under lower inflation rates which can dominate conventional debt deflation effects. Applying a calibrated version of the model, we find that particularly less indebted borrowers as well as potential future borrowers gain and that social welfare can be enhanced under a permanent reduction in inflation.

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1 Introduction

What are the effects of monetary policy under nominal rigidities in financial markets? Based on broad empirical evidence, the vast majority of studies on monetary policy effects considers nominal rigidities in goods and labor markets as the main sources of monetary non-neutrality. In contrast, the role of financial frictions has received much less attention in the literature, even though their existence can hardly be doubted. Debt is typically issued in nominal terms and in a non state-contingent way, such that changes in the price level can alter real payoffs. This transmission channel of unexpected monetary policy (the so-called Fisher debt deflation channel) is well-established and has been examined in several studies.[3] In this paper, we examine a novel counteracting channel of monetary transmission via a financial constraint, which is based on non-commitment of borrowers and the threat of repudiation.[4]

The central element of our analysis is that debt is limited by current income, which has repeatedly been reported by empirical studies (see e.g., Jappelli, 1990, Japelli and Pagano, 1989, Del Río and Young, 2006, Choi et al., 2015).[5] This observation can be rationalized by accounting for the inalienability of human capital (see Hart and Moore, 1994), here, of borrowers' labor (income). Suppose that borrowers do not hold any asset or durable good, such that debt cannot be collateralized. When borrowers' are not committed to repay debt, lenders' are threatened by the possibility that borrowers repudiate on debt contracts and divert their future labor income. In this case, lenders can take borrowers to court and seize up at most (a fraction of) current income. As a consequence, under a repudiation-proof debt contract debt repayment is restricted by current income; consistent with the empirical evidence.

In this paper, we explore implications for monetary policy and its redistributive effects under fully flexible goods prices when maximum debt repayment is constrained by current

[3] For example, Auclert (2016) or Doepke et al. (2015) are recent contributions to this literature. They further provide comprehensive overviews over studies on distributional effects of monetary policy.

[4] This for example differs from Sheedy (2014) who does not consider tight borrowing constraints, but where unexpected monetary policy can be non-neutral under aggeragte risk. Our mechanism is related to the friction introduced by mortgage contracts, as considered in Gariga et al. (2017).

[5] This observation has already been noticed by several studies accounting for debt limits being related to current income, like Laibson et al., (2003), Bianchi (2011), or Korinek (2017).

income.[6] Apparently, the debt limit in terms of commodities at maturity can then be affected by price level changes and thereby by monetary policy. To make this argument more transparent, consider a nominal repayment S_{t+1} that is contracted in t at the period t price Q_t and due in $t+1$. Suppose that it is limited at issuance by current income, $S_{t+1} \leq P_t y$, where y denotes an exogenous real income and P_t the price level in period t. Then, the real debt repayment in terms of commodities in period $t+1$, $x_{t+1} = S_{t+1}/P_{t+1}$, has to satisfy $x_{t+1} \leq y/\pi_{t+1}$, where π_{t+1} denotes the inflation rate $\pi_{t+1} = P_{t+1}/P_t$. Thus, a change in the inflation rate alters the effective debt limit, i.e. the maximum debt in terms of commodities at maturity.[7]

To understand the macroeconomic effects of monetary policy under a debt limit based on current income, consider for example an unexpected permanent increase in the inflation rate. A higher inflation rate implies the debt limit to shrink in terms of commodities at maturity. Given that this reduction in the price of debt at maturity is internalized by lenders, they also demand a lower debt price Q_t at issuance, which tends to reduce the maximum amount of funds that can be borrowed. The latter adverse effect is accompanied by the beneficial effect of the reduced debt repayment value in terms of commodities at maturity, which is in fact identical to the conventional debt deflation effect in the initial period. Hence, the increase in inflation tends to reduce the maximum amount of debt issued at the end-of-period (*debt limit effect*) as well as the beginning-of-period stock of debt to be repaid (*debt deflation effect*). The beneficial initial debt deflation effect is opposed to the adverse effect on the effective debt limit. Thus, the overall impact of higher inflation on borrowers' consumption possibilities and welfare is ex-ante ambiguous, and particularly depends on the likelihood that the borrowing constraint is binding and on the borrowers initial debt level. Moreover, when borrowing decreases due to a tighter effective debt limit under a higher inflation rate, the real interest rate tends to fall, which tends to lower the real cost of borrowing.[8]

[6] Note that such a constraint is not related to payment-to-income ratios that are relevant for mortgag (see Corbae and Quintin, 2015).

[7] If debt limits instead account for expected future price changes, debt limits would be fixed in terms of commodities at maturity, implying that monetary policy does not affect the effective tightness of the borrowing constraints.

[8] Notably, this pecuniary externality also applies (non-trivially) for the opposite case of lower inflation,

To assess the overall impact of changes in the inflation rate, we examine two distinct models. We first consider the highly stylized case of a stationary equilibrium of an economy where agents permanently differ by their degree of patience (as for example studied by Kiyotaki and Moore, 1997). Relatively impatient agents tend to frontload consumption and are willing to borrow from more patient agents up to the maximum amount. A higher inflation rate then leads to the opposed effects on borrowers described above, as debt repayment as well as the amount of newly issued debt are reduced. In this economy, where agents never switch types (borrower/lender), the beneficial deflation effect, in particular, on the initial stock of debt, dominates the debt limit effect, such that borrowers are better off with higher inflation rates. In contrast, if the borrowing limit were exogenously tightened, say, by an exogenous reduction in the fraction of current income, borrowers' welfare would not increase. The apparent reason is that this policy lacks the beneficial effect from a reduction of the initial debt burden, while it reduces initial and future consumption due to a tighter effective debt limit.

For the main part of our analysis, we focus on a second – less stylized – case and apply an incomplete market model (see Huggett, 1993) with nominal one-period debt and where agents differ with regard to their random individual income, while they are equally impatient.[9] When an agent draws a very low realization of income, he is willing to borrow up to the debt limit. The beneficial effect of a lower inflation rate, which tends to raise the effective debt limit and correspondingly the maximum amount of borrowed funds at issuance, might then outweigh the adverse debt deflation effect. This is actually the case when the probability of drawing again a low income level at maturity is sufficiently low, such that the marginal valuation of funds at issuance is sufficiently higher than the expected marginal valuation of funds at maturity. Put differently, a borrower tends to prefer a lower inflation rate and thus the relaxation of the effective debt limit even with

which increases the maximum debt repayment: Given that agents do not internalize how their demand for funds affects the real interest rate, a lower inflation rate can cause an increase in debt due to borrowers exploiting the higher debt limit, which tends to increase the real cost of borrowing. A related externality with regard to the real interest rate is discussed by Smith (2009).

[9]This set-up closely relates to Auclert's (2017) incomplete market model, which he uses to examine redistribution of monetary policy. In contrast to our model, the borrowing constraint in his model limits the amount of debt issued rather than outstanding debt, such that the changes in inflation does not alter the effective debt limit.

the higher debt repayment if he has a relatively high valuation of funds when debt is issued.[10]

We examine two versions of the incomplete market model with idiosyncratic risk, which mainly differ with regard to agents' utility function. For the first version, we assume that preferences are linear-quadratic and that income shocks ensure that the borrowing constraint always binds for borrowers, which facilitate aggregation and allow for the derivation of analytical results. Under these assumptions, the competitive equilibrium of the heterogenous agents economy can be characterized in terms of a representative borrower and a representative lender. For this economy, we show analytically that a reduction in the inflation rate can enhance welfare for the representative borrower if the autocorrelation of income shocks is sufficiently low, which tends to raise the gain from the debt limit effect relative to the debt deflation effect. The reason is that a constrained borrower is under a lower autocorrelation less likely to be again constrained at maturity, such that the expected marginal utility of consumption at maturity is reduced compared to the marginal utility of consumption at issuance. With this favorable effect for constrained agents, monetary policy can in fact enhance utilitarian social welfare by lowering inflation.

To assess the relevance of the beneficial effect of low inflation, we apply a second version of the model, imposing less restrictive assumptions. Specifically, we consider a standard CRRA utility function and a less stylized income process, and we account for the borrowing constraint to be occasionally binding. Given that this version cannot be solved analytically, we calibrate the model to match some characteristics of US postwar data and solve it numerically. Thereby, we find that compared with the calibrated model where the inflation rate equals 3%, the central bank can enhance utilitarian social welfare by reducing the average inflation rate to -3%. We find that borrowers with a high initial debt position suffer most from lower inflation, given that the debt deflation effect is dominant for them. In contrast, borrowers who are initially less indebted gain from lower inflation due to a dominant debt limit effect. Apparently, households with positive initial

[10] Studies on monetary policy in incomplete market economies with zero debt and fixed borrowing limits typically find effects of higher inflation rates that are beneficial for borrowers (see e.g. Akyol, 2004, Algan and Ragot, 2010, or Kryvtsov et al. 2011).

wealth in the period in which the central bank reduces the inflation rate benefit from both, the increase in initial real wealth and the higher debt limits in future periods in which they might be constrained. Given our calibration, social welfare increases with the reduction in the inflation rate, implying that the beneficial impact on the increase in the effective debt limit dominates the debt deflation effect in the aggregate.

In Section 2 we examine the redistributive effects of monetary policy in a stylized model with two agents which are characterized by different degrees of impatient. In Section 3 we apply a model where heterogeneity of agents, instead, originates from idiosyncratic income shocks. Section 4 concludes.

2 A model with patient and impatient agents

Before we examine financial frictions for monetary policy effects in a Huggett (1993) type model (see Section 3), we analyze the effects in a more stylized model to disclose the role of persistence of agents' types. Here, we assume that two types agents differ with regard to their degree of patience induced by different discount factors (as in Kiyotaki and Moore, 1997, or Eggertsson and Krugman, 2012). The patient agents with the higher discount factor will permanently be lenders and the impatient agents with the lower discount factor will permanently be borrowers. This will be the main difference between this model and the model in Section 3, where agents might switch roles in the credit market depending on their particular income draws and their endogenous net wealth positions.

2.1 The set-up

There is a continuum of infinitely lived agents of mass two, who have equal income from an exogenous labor supply, consume and trade one period nominal non-state contingent discount bonds at the issuance price $1/R_t$ $(= Q_t)$, paying one unit of currency in period t. For simplicity, we neglect uncertainty and disregard holdings of fiat money, which can be interpreted as the limit case of a cashless economy, while we assume it only serves as a unit of account (see also Sheedy, 2014, or Auclert, 2017). Households thus maximize the present value of future utilities $\sum_{t=0}^{\infty}(\beta_i)^t u(c_{i,t})$ with respect to a budget- and borrowing constraint, where $c_{i,t}$ is consumption of agent i and $i = l$ $(i = b)$ is the index of lenders (borrower), who constitute half of the population. The parameter β_i is the discount factor of agent i and satisfies $\beta_b < \beta_l < 1$. The utility function is identical for all agents and satisfies $u' > 0$ and $u'' < 0$. The nominal budget constraint of agent i is given by

$$P_t c_{i,t} = -(S_{i,t+1}/R_t) + S_{i,t} + P_t y_t, \tag{1}$$

where P_t denotes the price level, $S_{i,t}$ are nominal bond holdings with $S_{l,t} > 0$ and $S_{b,t} < 0$ and y_t is the endowment assumed to be identical for all. In each period, agents first trade in asset markets before they enter the goods market.

As central element of our analysis we consider that debt is restricted by current income, for which several studies found empirical support.[11] To rationalize this observation, we consider that agents cannot commit to repay debt and that human capital is inalienable (see Hart and Moore, 1994). Before the goods market opens, borrowers have the opportunity to repudiate on the debt contract by diverting future labor income, which can be observed by lenders. In this case, lenders can take borrowers to court and can seize current income. Due to imperfections in legal enforcement, we account that income cannot fully be seized, but only to a fraction $\gamma < 1$. Hence, a repudiation-proof debt contract restricts debt repayment to the fraction γ of current income. Finally, accounting for debt being specified in nominal terms, leads to the following borrowing constraint[12]

$$-S_{i,t+1} \leq \gamma P_t y_t. \tag{2}$$

In real terms, i.e. in terms of period t commodities, the real budget and borrowing constraints are given by $c_{i,t} = -s_{i,t+1}/R_t + s_{i,t}/\pi_t + y_t$ and $-s_{i,t+1} \leq \gamma y_t$, where $\pi_t := P_t/P_{t-1}$ denotes the inflation rate and $s_{i,t+1} := S_{i,t+1}/P_t$ the real value of wealth, which is a predetermined state variable in $t+1$. Accordingly, real end-of-period debt in the period of issuance t, $s_{i,t+1}$, is constrained by a fraction of real endowment in period t, y_t. Yet, when debt matures, prices might have changed, such that the real value $s_{i,t}$ has to be adjusted by the inflation rate to account for real debt burden in term of commodities at maturity, i.e. $s_{i,t+1}/\pi_{t+1} = S_{i,t+1}/P_{t+1}$. Accordingly, the borrowing

[11] Examples are Jappelli (1990), Japelli and Pagano (1989), Del Río and Young (2006) or Choi et al. (2015).

[12] Theoretical studies where borrowing is also constrained by current income, are for example Laibson et al., (2003), Bianchi (2012), or Korinek (2017).

constraint $-S_{i,t+1}/P_{t+1} \leq \gamma y_t/\pi_{t+1}$ shows that the inflation rate determines the limit for debt repayment in terms of commodities at maturity $t+1$. Hence, a higher inflation rate decreases real debt repayment at maturity of borrowers. Maximizing lifetime utility, the borrowers' and lenders' first order conditions, which describe their optimal allocation of consumption over time, are given by

$$u'(c_{b,t})/R_t = \beta_b u'(c_{b,t+1})\pi_{t+1}^{-1} + \zeta_{b,t}, \tag{3}$$

$$u'(c_{l,t})/R_t = \beta_l u'(c_{l,t+1})\pi_{t+1}^{-1}, \tag{4}$$

where $\zeta_{b,t}$ denotes the multiplier on the borrowing constraint (2), which is irrelevant for lenders. Further, the associated complementary slackness conditions hold.

In this cashless economy, the central bank can control the nominal interest rate via a channel system. Given that changes in the nominal interest rate will affect the (expected) inflation rate, we will assume, for convenience, that the central bank controls the inflation rate by setting the interest rate in order to meet specific inflation targets (see also Sheedy, 2014). Given that there is no aggregate uncertainty, we will focus on constant inflation targets, $\pi > 0$. Notably, the inflation choice might imply values for the nominal interest rate for which the zero lower bound, $R_t \geq 1$, is binding.

The equilibrium is then a set of sequences $\left\{c_{b,t}, c_{l,t}, s_{b,t+1}, s_{l,t+1}, R_t, \zeta_{b,t} \geq 0\right\}_{t=0}^{\infty}$ for a given constant inflation rate $\pi > 0$ and a given constant endowment $y_t = y > 0$ satisfying (3) and (4), $c_{b,t} = -(s_{b,t+1}/R_t) + (s_{b,t}/\pi) + y$, $c_{b,t} + c_{l,t} = 2y$, $-s_{b,t+1} \leq \gamma y$ and $-s_{b,t} = s_{l,t}$ together with the complementary slackness condition, given $-s_{b,0} = s_{l,0}$. The real interest rate satisfies $R_t/\pi_{t+1} = \beta_l u'(c_{l,t+1})/u'(c_{l,t})$ and will be strictly positive in a long-run equilibrium (see 4), i.e. it equals the inverse of the lenders' discount factor , $R/\pi = 1/\beta_l > 1$. Given that $\beta_b < \beta_l$, the borrower will be constrained in a long-run equilibrium, $\zeta_b > 0$ (see 3).

2.2 Results

We now examine the effects of a permanent change in the inflation rate in this simple economy. For a given initial value of real wealth $s_{b,0} = S_{b,0}/P_{-1}$, a permanent change in the inflation rate is equivalent to an unexpected permanent inflation shock in period $t = 0$

where the borrower enters period $t = 0$ with beginning-of-period wealth $s_{b,0}$. Suppose that the latter is sufficiently close to its steady state value, such that the economy will be in the steady state in period $t \geq 1$. Using the steady state real interest rate, $R/\pi = 1/\beta_l$, and that borrowers are always constrained, $s_{b,t} = -\gamma y$, the borrowers' budget constraint, $c_{b,t} = -s_{b,t+1}R_t^{-1} + s_{b,t}\pi_t^{-1} + y$, implies steady state consumption (in $t \geq 1$) and initial consumption to satisfy

$$c_{b,t} = -\gamma(y/\pi)(1-\beta_l) + y, \ \forall t \geq 1, \tag{5}$$

$$c_{b,0} = \gamma y/R_0 - s_{b,0}/\pi + y, \tag{6}$$

where for initial consumption $c_{b,0}$ the beginning of period stock of debt, s_0, is given. Since $\beta_l < 1$, which implies a positive real interest rate, borrowers' consumption for $t \geq 1$ is increasing in the inflation rate (see 5). On the one hand, a higher inflation rate decreases the real value of outstanding debt $s_{b,t}/\pi$ due to the debt deflation effect, which tends to increase consumption. On the other hand, a higher inflation rate lowers the debt limit in terms of commodities at maturity. Likewise, the issuance price of debt adjusts according to lenders willingness to supply funds (4), such that the nominal interest rate tends to increase. Hence, the maximum amount of funds that can be borrowed in the steady state, $s_b R^{-1}$, declines, which tends to decrease consumption of the constrained borrower. In the steady state, the constrained borrower has to roll over debt at a positive real interest rate larger than one, such that higher inflation increases borrowers' consumption by reducing the costs of debt that has to be rolled over (see 5 and 6). While this results seems to be fairly intuitive, two points should be noted here.

1. A higher inflation rate would have no effect on borrowers' consumption in periods $t \geq 1$, if the borrowing limit were specified terms of commodities in period $t+1$. If borrowing were instead limited by $-S_{i,t+1} \leq \gamma P_{t+1} y_t \Longleftrightarrow -s_{i,t+1}/\pi_{t+1} \leq \gamma y_t$, borrowers' consumption in $t \geq 1$ (in which the economy is in a steady state) would be given by $c_b = -\gamma y (1-\beta_l) + y$. Monetary policy would then be neutral in the steady state, as the debt limit is not affected by price changes.

2. A permanent reduction in the debt-to-income ratio γ starting in period $t = 0$,

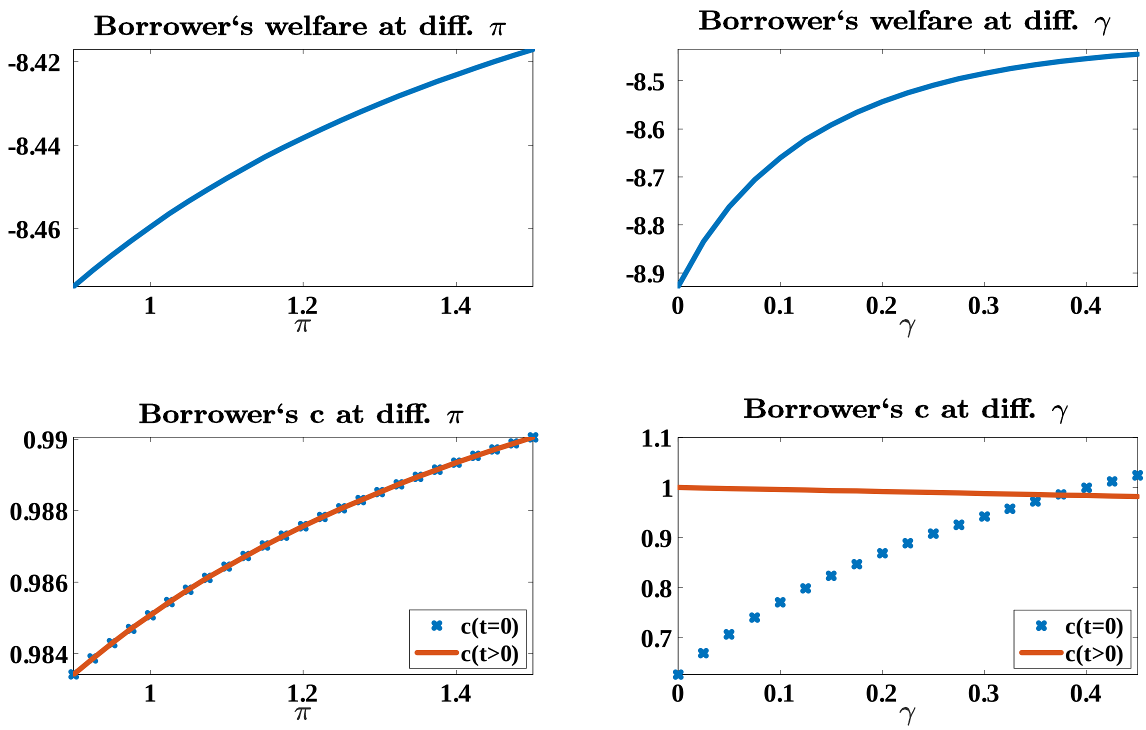

Figure 1: Consumption and welfare of relatively impatient borrowers

which for example might be imposed by regulation, is actually not equivalent to an increase in the inflation, which can be seen from (6). The reason is that a change of γ in $t = 0$ cannot affect the real initial wealth $s_{b,0}$ (for which γ would already have to be changed in $t = -1$).

We now assess the impact of inflation via the two channels (the debt deflation channel and the debt limit channel) described above. To abstract from transitional dynamics, we assume that the borrower is initially endowed with the steady state stock of debt, $s_{b,0} = -\gamma y$. Notably, the latter assumption implies that the borrower will be in a steady state in all periods $t \geq 0$ with $s_{b,t} = -\gamma y$ regardless of the inflation rate. Figure 1 shows the steady state effect of the inflation rate π and the debt to income ratio γ on borrower's consumption and welfare, which is given by $\sum_{t=0}^{\infty} \beta_b^t u(c_{b,t}) = u(-\gamma(y/\pi)(1-\beta_l)+y)/(1-\beta_b)$. The chosen parameter values are $y = 1$, $\beta_l = 0.96$, $\beta_l = 0.88$ and $\gamma = 0.37$ with a CRRA utility function $u(c_i) = c_i^{1-\sigma}/(1-\sigma)$ and $\sigma = 2$ (see Section 3.2.2 for a discussion of the parameter values). The first column shows the effect of a change in the inflation rate at a constant value of γ. The second column shows the effect of a change in the fraction γ

at a constant inflation rate $\pi = 1$. The first column shows that consumption and welfare of borrowers unambiguously increase with the inflation rate, in accordance with effects described above. The second column of Figure 1 displays the effects of changes in the fraction γ. A reduction in the latter has a positive impact on borrowers' consumption in $t \geq 1$, like a higher inflation rate (see solid line). In contrast to a higher inflation rate, a lower value γ has an adverse effect on borrowers' initial consumption, since initial debt $s_{b,0}$ is given (see 6). The decline in initial consumption dominates the beneficial effects in the subsequent periods, such that borrowers' welfare decreases with a tighter borrowing constraint induced by a lower fraction γ.

3 A model with idiosyncratic risk

In this Section, we apply a Hugget-type model, where we neglect differences in the discount factor. Idiosyncratic endowment shocks induce agents to borrow/lend, while there is no aggregate risk. As in the model presented in the previous section, only non-state-contingent nominal debt is available such that agents cannot share risk.

3.1 The set-up

Consider an economy with infinitely lived and infinitely many households i of mass two. These households share the same utility function, but might differ with regard to a random idiosyncratic nominal income $Y_{i,t}$. Preferences of a household i are given by

$$E \sum_{t=0}^{\infty} \beta^t u(c_{i,t}), \tag{7}$$

where E denotes an expectations operator and $c_{i,t}$ consumption of household i. As before, the utility function $u(c_{i,t})$ is assumed to satisfy $u' > 0$ and $u'' < 0$. Note that below we will examine the model for two different types of utility functions. First, we apply a linear-quadratic (LQ) utility function, which facilitates aggregation and the derivation of analytical results. Second, we use a standard CRRA utility function for a numerical analysis.

Real income $y_i = Y_i/P$ is identically and independently distributed over all households, but might be serially correlated over time. We consider a finite set of n possible realizations of the random variable y, y_1,... y_n with transition probabilities $p_{k,l}$ from state k to state l and a positive unconditional mean $Ey_i = \overline{y} > 0$. When a household draws $y_i < y_{i+1}$ as a realization of the income shock, it has less funds available and tends to borrow from other households with a larger income. Shocks are realized at the beginning of each period, before the asset market opens. Once, these shocks are realized, households enter the asset market where they repay debt and can borrow/lend funds from/to other households. Accounting for idiosyncratic income y_i and budget constraint and the borrowing constraint are now given by

$$c_{i,t} + s_{i,t+1}/R_t \leq s_{i,t}\pi_t^{-1} + y_{i,t}, \tag{8}$$

$$-s_{i,t+1} \leq \gamma y_{i,t}. \tag{9}$$

To disclose the main mechanism, we will also apply a simplified borrowing constraint for the derivation of analytical results, $-s_{i,t+1} \leq b$, where the constant b can be interpreted as referring to mean income, $b = \gamma\overline{y}$ (see Section 3.2.1). Households aim at maximizing lifetime utility (7) subject to (8) and (9) taking prices as given. Households who draw an income shock y_i tend to borrow from households who draw $y_j > y_i$. Household $i's$ first order conditions for a given initial endowment $s_{i,0} = 0$ are $\lambda_{i,t} = u'_{i,t}$,

$$u'_{i,t}/R_t = \beta E_t[u'_{i,t+1}\pi_{t+1}^{-1}] + \zeta_{i,t}, \tag{10}$$

where $\lambda_{i,t}$ and $\zeta_{i,t} \geq 0$ denote the multiplier on (8) and (9), respectively. Further, the budget constraint (8) is binding and the complementary slackness conditions for (9), $0 = \zeta_{i,t}s_{i,t+1}$, and $\zeta_{i,t} \geq 0$, hold.

In equilibrium, prices adjust such that plans are realized and markets clear. A competitive equilibrium is a set of sequences $\{c_{i,t}, s_{i,t+1}, w_t, R_t, \zeta_{i,t}\}_{t=0}^{\infty}$ satisfying (8)-(10), $y_t = \Sigma_i y_{i,t} = \Sigma_i c_{i,t}$ and $\Sigma_i s_{i,t+1} = 0$ the complementary slackness conditions, for a given inflation rate π and given $s_{i,0}$. The first best allocation $\{c^*_{i,t}\}_{t=0}^{\infty}$ evidently satisfies $u'(c^*_{i,t}) = u'(c^*_{j,t})$ for all agents $i \neq j$, which we will consider as a benchmark case.

3.2 Results

In this section, we examine the redistributive consequences of changes in the monetary policy instrument, i.e. inflation, and the effects on utilitarian social welfare. This model can in general not be solved analytically, given that agents might have different histories of $y_{i,t}$-draws and their decisions depend on their beginning-of-period net wealth $s_{i,t}$. We will therefore apply some simplifying assumptions regarding preferences and shocks in the first part of the analysis. In the second part, we apply numerical methods for a less simplified version of the model to assess the robustness of the main results.

The analysis in the previous section suggests that borrowers gain from higher inflation. However, it turns out that this might not be the case, in particular, when agents – in contrast to the model in Section 2 – change their roles in the credit market and borrowers have a low initial stock of debt. In this case, a borrower can actually gain from a low inflation rate, which increases the effective debt limit and thus tends to increase the amount of funds borrowed, such that a borrower might prefer low rather than high inflation rates. This however depends on the relation between the marginal utilities at issuance and when debt matures. A borrower who is constrained in the issuance period, but is unconstrained at maturity with a positive probability, can in fact gain from a reduction in the inflation rate even when it is accompanied by a higher real debt repayment.

To make these ambiguous effects of monetary policy transparent, we apply in Section 3.2.1 some simplifying assumption, which allow to derive analytical results. Specifically, we consider a constant borrowing limit, a linear-quadratic utility function and we assume that the borrowing constraint binds for agents who draw a low income level, which greatly facilitates aggregation. In Section 3.2.2, we present quantitative results for which we drop the latter assumptions and apply the borrowing limit (9) as well as a standard CRRA utility function.

3.2.1 A version with representative borrower and lender

In this subsection, we analytically examine the main effects of monetary policy in the economy with idiosyncratic income shocks. We consider two realizations for income, y_1

and y_2, and assume that the transition probabilities of income shocks satisfy $p_{12} = p_{21}$ and $p_{11} = p_{22}$. To derive analytical results, we impose the assumption of a constant borrowing limit and a linear-quadratic utility function (which might be interpreted as a second order approximation of an original utility function).

Assumption 1 *The borrowing constraint is given by $-s_{i,t+1} \leq b$ and households' preferences satisfy $u(c_{i,t}) = (\delta c_{i,t} - c_{i,t}^2)$,* where $\delta > y_1 + y_2 = 2\overline{y}$.

When preferences satisfy Assumption 1, the marginal utilities are obviously linear in individual consumption, which greatly facilitate aggregation over individual household choices. We further restrict our attention to the case where the variance of the preference shocks are sufficiently large such that the borrowing constraint will always be binding for agents drawing y_1. To achieve this, we apply a relatively large difference of $y_2 - y_1$ compared to the parameter b governing the tightness of the borrowing constraint.

Assumption 2 *The ratio $b/(y_2 - y_1)$ is sufficiently small such that $\zeta_{j,t} > 0$ for all households j drawing y_1.*

Hence, for a borrowing agent the end-of-period net-wealth positions will equal $-b$. Accordingly, lenders, which are of the same mass as borrowers, can only have a wealth position equal to (minus) the debt level of borrower. As for the model with different degrees of patience, we analyze the effects of inflation on agents initially endowed with $s_{i,0} = -b$ or $s_{i,0} = b$ and $\Sigma_i s_{i,0} = 0$ to abstract from transitional dynamics.

Aggregation We aim at aggregating over individual choices of agents with the borrowing constraint and preferences satisfying Assumption 1 and shocks satisfying Assumption 2. For this, we separately analyze two types of agents, borrowers drawing a_1 and potential lenders drawing a_2. The choices of the former are characterized by the conditions $(\delta - 2c_{(b,i),t})/R_t = (\beta/\pi)\left[p_{11}(\delta - 2c_{(b,i),t+1}) + p_{12}(\delta - 2c_{(l,i),t+1})\right] + \zeta_{(b,i),t}$, $-s_{(b,i),t+1} \leq b$, and $c_{(b,i),t} = -s_{(b,i),t+1}R_t^{-1} + s_{(b,i),t}\pi^{-1} + y_1$, where $\zeta_{(b,i),t} \geq 0$ and $\zeta_{(b,i),t}b = 0$. Given that all conditions are linear in the choice variables for $\zeta_{(b,i),t} > 0$, we can easily aggregate. Let $c_{b,t} = \Sigma_{b,i}c_{(b,i),t}$, $\zeta_{b,t} = \Sigma_{b,i}\zeta_{(b,i),t}$ and $s_{b,t+1} = \Sigma_{b,i}s_{(b,i),t+1}$. Then, we get the following set of conditions describing the behavior of a representative borrower:

$$(\delta - 2c_{b,t})/R_t = (\beta/\pi)\left[p_{11}(\delta - 2c_{b,t+1}) + p_{12}(\delta - 2c_{l,t+1})\right] + \zeta_{b,t}, \tag{11}$$

$$-s_{b,t+1} = b, \tag{12}$$

$$c_{b,t} = -(s_{b,t+1}/R_t) - p_{11}(b/\pi) + p_{21}(b/\pi) + y_1, \tag{13}$$

and $\zeta_{b,t} > 0$. Note that we used that beginning of period wealth either equals b or $-b$, depending on whether the current borrower was a lender or a borrower in the previous period. Using the law of large numbers, a fraction of p_{11} (p_{21}) of previous borrowers (lenders) draw y_1 in the current period. Thus, current period initial net wealth of the representative borrower equals the weighted average $-p_{11}(b/\pi) + p_{21}(b/\pi)$. Apparently, the same arguments apply for all agents drawing y_2, such that we can proceed analogously and get the following conditions describing the behavior of a representative lender:

$$(\delta - 2c_{l,t})/R_t = (\beta/\pi)\left[p_{21}(\delta - 2c_{b,t+1}) + p_{22}(\delta - 2c_{l,t+1})\right], \tag{14}$$

$$c_{l,t} = -(s_{l,t+1}/R_t) - p_{12}(b/\pi) + p_{22}(b/\pi) + y_2, \tag{15}$$

We thus characterize a competitive equilibrium in terms of a representative borrower and lender.

Definition 1 *A competitive equilibrium of the representative agents economy is a set of sequences* $\{c_{b,t},\, c_{l,t},\, s_{b,t+1},\, R_t,\, \zeta_{b,t} > 0\}_{t=0}^{\infty}$ *satisfying (11), (12), (14),*

$$c_{b,t} - c_{l,t} = -(2s_{b,t+1}/R_t) - (p_{11} - p_{21})(b/\pi) + (p_{12} - p_{22})(b/\pi) + y_1 - y_2, \tag{16}$$

$$c_{b,t} + c_{l,t} = a_1 + a_2 \tag{17}$$

for a given inflation rate $\pi > 0$.

Next, we want to examine policy choices for a competitive equilibrium in terms of a representative borrower and a representative lender. For this, we assume that the social planer aims at maximizing utilitarian social welfare, i.e. the sum of welfare of the representative borrower and of the representative lender.

Effects of monetary policy We now want to examine how monetary policy affects the allocation and social welfare in the representative agents economy given in definition

1. Specifically, we analyze the effects of the inflation rate and identify the monetary policy that maximizes utilitarian social welfare. For this, we use that by Assumption 2 consumption of the representative borrower is smaller than consumption of the representative lender due to the binding borrowing constraint. Thus, marginal utility of the representative borrower is larger than the marginal utility of the representative lender, $(\delta - 2c_{b,t}) > (\delta - 2c_{l,t})$, for which we combined the conditions (11) and (14). Hence, a redistribution of consumption from the latter to the former can be welfare improving. It can be shown that this can be induced by reducing the nominal interest rate and thereby the inflation rate if the serial correlation of endowment shocks is not too high.

Proposition 2 *Consider a competitive equilibrium as given in Definition 1. A reduction in the inflation rate is welfare improving if* $p_{12} > (1-\beta)/2$*. Monetary policy is then able to implement first best if* $1 \leq 2b[1 + (p_{21} - p_{11})/\beta]/(y_2 - y_1)$*. Otherwise, the zero lower bound on the nominal interest rate prevents monetary policy from implementing first best.*

Proof. We start by establishing the first claim of the proposition. By Assumption 2 we have $c_1 < c_2$, such that an increase in c_1 and thus a decrease in c_2 by the same amount causes a reduction of the gap between the marginal utility of the representative borrower and the marginal utility of the representative lender. Under a constant inflation rate, the equilibrium exhibits no time variation, such that we can neglect time indices. Substituting out the interest rate with (14), which can – by using (17) – be rewritten as

$$1/R = (\beta/\pi)\,[p_{21}\frac{\delta - 2c_b}{\delta + 2c_b - 2(y_1 + y_2)} + p_{22}], \tag{18}$$

in the borrower's budget constraint (13), gives

$$c_b = (b/\pi)\left(\beta p_{21}\frac{\delta - 2c_b}{\delta + 2c_b - 2(y_1 + y_2)} + \beta p_{22} + p_{21} - p_{11}\right) + y_1,$$

where the fraction on the RHS is strictly decreasing in c_b. Thus, a lower inflation rate increases c_b if the term in the round brackets is positive, i.e.

$$\beta\left[p_{21}\frac{\delta - 2c_b}{\delta + 2c_b - 2(y_1 + y_2)} + p_{22}\right] + 2p_{21} - 1 > 0, \tag{19}$$

where we used $p_{21} + p_{11} = 1$ for a symmetric transition matrix. The term in the squared brackets is larger than one under a binding borrowing constraint, since $p_{21} + p_{22} = 1$ and the marginal utility of the representative borrower is larger than marginal utility of the representative lender implying $\frac{\delta-2c_1}{\delta+2c_1-2(y_1+y_2)} > 1$. Thus, $\beta + 2p_{21} - 1 > 0$ is sufficient to satisfy the inequality (19). In this case, a lower inflation rate increases c_b and, therefore, utilitarian welfare.

To establish the remaining claims of the proposition, we use that $c_{b,t} = c_{l,t}$ holds under first best, (16) implying

$$-s = R(y_2 - y_1 + 2(b/\pi)(p_{11} - p_{21}))/2 \leq b,$$

where in inequality is due to the non-binding borrowing constraint under first best. Under first best, (14) further implies $R/\pi = 1/\beta$. Substituting out inflation with the latter, we can rewrite the inequality as

$$R \leq 2b\frac{1 + (p_{21} - p_{11})/\beta}{y_2 - y_1}.$$

The last inequality and the ZLB imply $1 \leq 2b\frac{1+(p_{21}-p_{11})/\beta}{y_2-y_1}$ for monetary policy to be able to implement first best. If however $1 > 2b\frac{1+(p_{21}-p_{11})/\beta}{y_2-y_1}$ monetary policy cannot implement first best due to the ZLB. ■

According to proposition 2, monetary policy should choose a low inflation rate to maximize social welfare if the probability of changing income types is sufficiently large, i.e. $p_{21} = p_{12} > (1 - \beta)/2$. The reason for this result is that inflation exerts two opposing effects on borrowers and in particular on their consumption level.[13] Under the borrowing constraint in terms of commodities at issuance, $-s_b \leq b$ (as assumed here), the amount of funds that can be issued b/R and the repayment b/π decrease with the inflation rate. Thus, monetary policy is non-neutral, while its overall impact on borrowers depends on the subjective valuation of funds at issuance and at maturity, which depends on the marginal utility of consumption. Consider for example a household who draws y_1 today and borrows funds up to the borrowing constraint. If the probability of being unconstrained at maturity is positive, its expected marginal utility then tends

[13]If borrowing were constrained by a debt limit in terms of commodities at maturity, i.e., $-s_b/\pi \leq b$. If its is non-binding, first best again realizes. If it is binding, the amount that has to be repaid simplify equals b, such that monetary policy is neutral.

to be lower than today. This household would gain from a proportional reduction in the nominal interest rate, which raises the amount of funds that can be borrowed today, even if its is accompanied by a proportional reduction in the inflation rate, which tends to raise real debt repayment. Thus, under a sufficiently large probability of drawing a high income shock and being unconstrained at maturity the debt limit effect dominates the debt deflation effect, such that monetary policy should lower rather than raise inflation to benefit borrowers. This result is consistent with the findings in Section 2, where borrowers remain always remain constrained and where they gain from higher inflation.

Though, the condition for lower inflation to enhance welfare, i.e. $p_{21} = p_{12} > (1-\beta)/2$, seems to be fairly week (given that discount factors are typically close to one), it remains to assess whether the arguments made are of quantitative relevance. For this, we apply a calibrated version of the model, for which less restrictive assumptions are made than in this subsection.

3.2.2 A calibrated version

The previous analysis has shown that monetary policy can enhance welfare by reducing inflation and the nominal interest rate, if borrowing agents are less likely to be constrained when they repay debt. Yet, this analysis has been conducted under extreme assumptions on preferences, the debt limit, and shocks. Here, we examine a less stylized framework, for which we omit the simplifying Assumptions 1 and 2. Instead, we analyze the debt limit and the debt deflation effect in a model version, which is calibrated applying US data. Instead of Assumption 1, we apply the borrowing limit (9) and a conventional CRRA period utility function for households $i \in [0, 1]$

$$u(c_{i,t}) := c_{i,t}^{1-\sigma}/(1-\sigma), \tag{20}$$

where $\sigma > 0$. In contrast to Assumption 2, we further allow for the borrowing constraint not to be binding for borrowers. As a consequence, individual wealth/debt of agents can vary over time depending on the individual history of endowment shocks. The realizations of these shocks are now assumed to satisfy $y_{i,t} \in \{y_1, y_2, ..., y_n\}$, where $0 < y_j < y_{j+1}$ for $j = 1, ..., n-1$, and to follow a first-order Markov process with transition matrix Q, which is identical for all households, given by $Q_{k,l} := p_{k,l}$ for $k, l = 1, ..., n$, where $p_{k,l}$ is the probability to switch from state k in $t-1$ to state l in period t.

The equilibrium conditions for a household i in income state $y_{i,t} = y_j$ for $j = 1, ..., n$ and wealth state $s_{i,t} = s_t$ are now given by

$$c_t(s_t, y_j) = -s_{t+1}(s_t, y_j)R_t^{-1} + s\pi_t^{-1} + y_j, \tag{21}$$

$$c_t(s_t, y_j)^{-\sigma}/R_t = (\beta/\pi_{t+1}) \sum_{l=1}^{n} p_{j,l} c_{t+1}(s_{t+1}(s_t, y_j), y_l)^{-\sigma} + \zeta_t(s_t, y_j), \tag{22}$$

$$-s_{t+1}(s_t, y_j) \leq \gamma y_j, \tag{23}$$

$$\zeta_t(s_t, y_j) \geq 0, \tag{24}$$

and the complementary slackness condition for a given s_0.

In the following we aim at examining the effects of the following policy experiment. We assume that the economy is in the stationary equilibrium induced by the benchmark inflation rate of 3% at period t. We then examine the effects of an unexpected permanent reduction in the inflation rate to -3% in period $t+1$ on the allocation and social welfare. After the change in inflation, the economy leaves the stationary equilibrium induced by an inflation rate of 3% and converges to the new one under the lower rate of -3%. Therefore, we first calculate the stationary equilibrium for both inflation rates. Notably, this choice guarantees that the nominal interest satisfies the zero lower bound in the new steady state.

Let λ be a distribution of agents, where $\lambda(s, y)$ is the measure of agents with wealth s and income y. The stationary equilibrium then consists of a nominal interest rate R, constant policy functions $c(s, y)$ and $s'(s, y)$ and a distribution $\lambda(s, y)$ consistent with a particular constant monetary policy π such that 1) decision rules solve the individual optimization problem, 2) markets clear $\sum_{s,y} \lambda(s, y)c(s, y) = \sum_{s,y} \lambda(s, y)y$ and $\sum_{s,y} \lambda(s, y)s'(s, y) = 0$, and 3) $\lambda(s, y)$ is time invariant (see Appendix A.2). Having constructed the stationary equilibria we calculate the transition path from the old to the new stationary equilibrium (see Appendix A.3). Note that the policy functions, wealth distribution and nominal interest rate are not constant over time anymore after the change in the inflation rate but then converge to the time invariant functions and values of the stationary equilibrium under the new inflation rate of -3%.

Calibration The model further contains four parameters, namely, the degree of relative risk aversion σ, the subjective discount factor β, the fraction γ, and the moments of the idiosyncratic income process. The length of a period is assumed to equal 1 year. While σ is set at 2 in accordance with many related studies, the parameters β and γ are set to roughly match empirical targets. Specifically, they are set at $\beta = 0.92$ and $\gamma = 0.37$ to get a real annual risk free interest rate of 4.06% and a debt to income ratio of about 16%; the latter being close to the empirical counterparts of 14% for US postwar data for installment loans and after tax income in 2004 taken from the CBPP "New CBO Data Show Income Inequality Continues to Widen" (www.cbpp.org/research/new-cbo-data-show-income-inequality-continues-to-widen) and the Survey of Consumer Finances.

For the income process, we assume that log individual annual income follows an AR1 process, $\ln(y_{i,t}) = \rho \ln(y_{i,t}) + \epsilon_{i,t}$, with $\epsilon_{i,t}$ i.i. normally distributed with mean 0, variance σ_ϵ^2. We apply the income states $y_1 = 0.81$, $ya_2 = 0.89$, $y_3 = 1.08$, $y_4 = 1.18$ and $y_5 = 1.54$. For the benchmark parametrization, we use Storesletten et al.'s (2004) estimates for the autocorrelation coefficient and the variance, $\rho = 0.963$ and $\sigma_\epsilon^2 = 0.13^2$. We then apply Tauchen's (1986) algorithm for the five states of the log-labour-income process, which leads to the transition matrix Q given in Appendix A.1 and a stationary distribution of 20% in every income state. Given these parameter values, we compute the solution of the model applying an endogenous grid point method to calculate the stationary equilibrium (see Appendix A.2).

To see how the model implied distribution of debt for a benchmark inflation rate of 3% relates to its empirical counterpart based on installment loans, Figure 2 shows the untargeted debt-to-income ratios for the five income states and the empirical debt-to-income ratios of 2004 based on installment loans, which correspond most closely to the unsecured loans in the model. The model is actually able to fit the debt-to-income ratio for the income quintiles reasonably well. Notably, the model underestimates the debt to income ratios for the highest income. The reason is that households in the highest income quintile have a relatively low incentive to borrow in our model, as they tend to save for consumption smoothing.

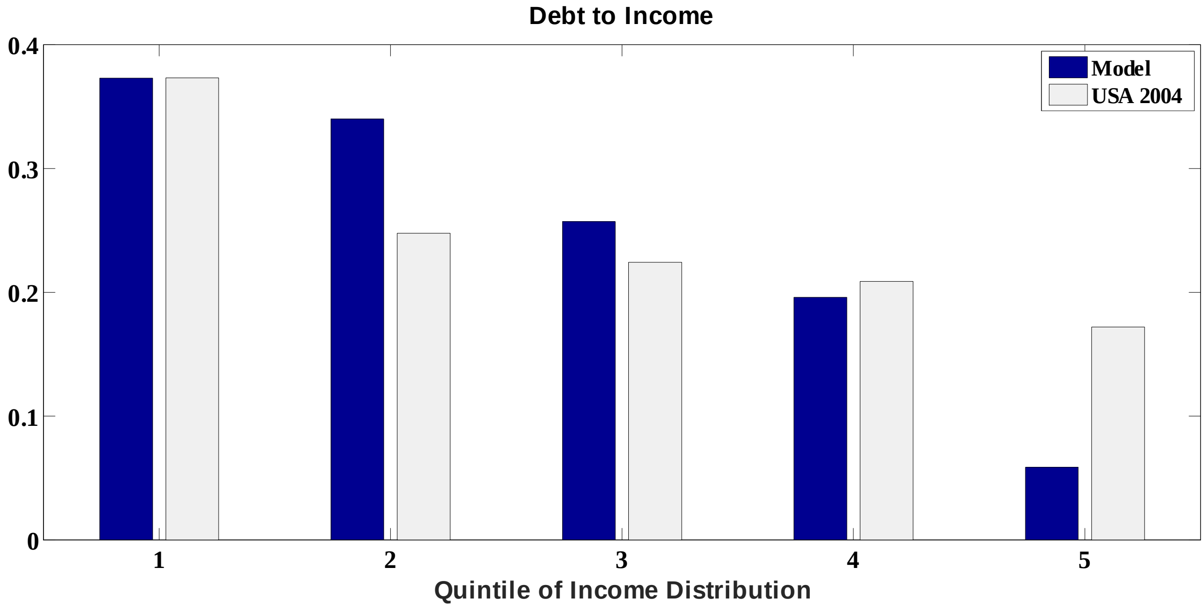

Figure 2: Debt-to-income ratio for US income quintiles (model: dark blue bars, data: light grey bars)

How does inflation affect agents' choices? We first examine the effects of a change in the inflation rate on consumption and savings in the initial period ($t = 0$). Assume that the distribution of predetermined wealth s_0 at time 0 is given by the stationary distribution induced by an inflation rate of 3%. Then at time 0 monetary policy unexpectedly decreases the inflation rate to -3% and holds it constant at -3% thereafter. As mentioned above, the economy leaves the old stationary equilibrium under 3% inflation and converges to the new one under -3%. To calculate the transition path we first compute the old and the new stationary equilibrium (see Appendix A.2). We assume that the economy reaches the new stationary equilibrium after T periods and then calculate the path for the nominal interest rate such that the corresponding policy functions imply a path for the wealth distribution that converges to the wealth distribution of the new stationary equilibrium after T periods (see Appendix A.3). The reduction in inflation has no impact on the distribution of predetermined wealth s_0. Yet, it does change the effective initial net wealth, i.e., the real value of initial net wealth in terms of current period commodities, $s_0/\pi_0 = S_0/P_0$. Via the debt deflation effect, the lower inflation rate increases both the initial net wealth/debt as well as all future debt repayments in terms of commodities at maturity for borrowers who are constrained at issuance. Via the debt

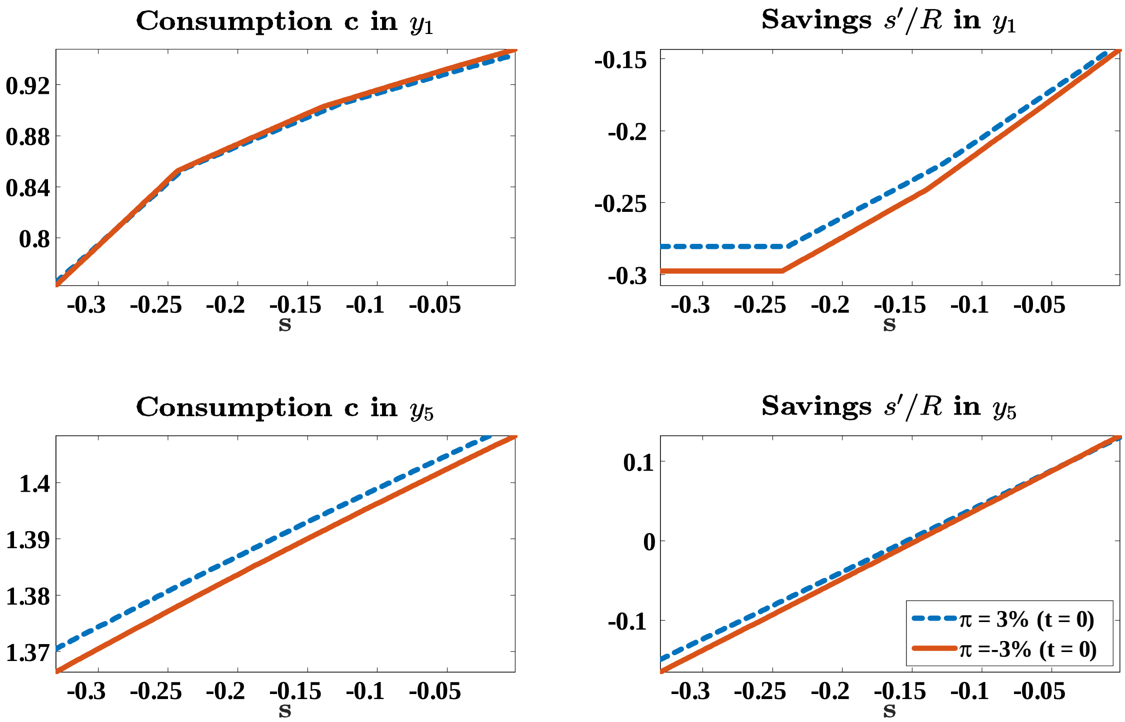

Figure 3: Policy functions for consumption and savings in period 0 for 3% inflation (blue dashed line) and for -3% (red solied line)

limit effect, lower inflation raises the issuance price of debt and the maximum amount of funds that can be borrowed. Whether lower inflation is beneficial or not for a borrower who is constrained at issuance depends – inter alia – on the likelihood to be again constrained at maturity. Remember that the initial debt deflation effect has dominated the welfare effect in the model with different degrees of patience, where initial borrowers are constrained in all periods (see Section 2). In contrast, the debt limit effect can dominate in the economy with idiosyncratic shocks if borrowers who are constrained at issuance have to repay the higher debt obligations while being unconstrained at maturity with a positive probability.

To understand the effects on the allocation and on social welfare, and to compare them with the results obtained in the model of Section 2, we first examine policy functions for consumption and savings for the given net wealth distribution in period 0. Specifically, we apply the policy functions for consumption $c(s, y)$ and savings $s'(s, y)/R$ for different income states of both the economy under a constant inflation rate of 3% and the economy under the lower inflation rate of -3%. The reduction in the inflation rate reduces the net nominal interest rate from 7.18% to 0.98% in period 0, which then converges to 1.06%; the latter being the value for the stationary distribution induced by the lower inflation rate of -3%. The reduction in the inflation rate implies an increase in the effective limit for borrowed funds, $s'(s, y)/R \leq \gamma y/R$. To clear the market, this however requires a higher real interest rate, which under our calibration increases the real interest rate (slightly) from 4.06% to 4.10%. The corresponding policy functions (see Figure 3) show consumption and savings for the case of 3% inflation and for the experiment of an unexpected reduction in inflation to -3% in the initial period $t = 0$. For convenience, we focus on the incomes states a_1 and a_5 and on initially indebted agents ($s_0 < 0$), while a corresponding figure that also includes agents with positive net initial wealth can be found in Appendix 3.2.2 (see Figure 7).

Intuitively, the reduction in the inflation rate increases the value of initial effective debt (positive wealth) and thereby tends to decrease (increase) consumption as in the model without idiosyncratic shocks. The policy functions in Figure 3 show that constrained borrowers (see horizontal lines in the upper right panel) in the income state y_1 with relatively high initial debt slightly decrease consumption in the initial period due to the debt deflation effect (see also Section 2). However, the initial debt deflation effect is not dominant for all initially indebted households (see upper left panel). Firstly, constrained borrowers with relatively low initial debt tend to raise consumption by increasing borrowing. Thus, for relatively low initial debt obligations, the debt limit effect dominates the initial debt deflation effect. Secondly, consumption under low inflation is also higher for unconstrained borrowers with initial debt in y_1, because these households, who have a relatively high probability to be constrained in future periods, can potentially increase borrowing due to higher effective debt limits in the future (see 3). Put differently, their precautionary savings motive is less pronounced due to an improved access to external funds. Moreover, lower inflation tends to increase consumption more for positive initial net wealth levels s_0 (see first row of Figure 7 in the Appendix) due to the debt deflation effect. For the highest income state y_5, for which consumption and savings are shown in the second row of Figure 7, agents are not constrained and the adverse debt deflation effect of lower inflation dominates. Finally, it should be noted that the policy functions under a stationary wealth distribution for an inflation rate of -3% are virtually identical to the policy functions in period 0 (see Figure 8 in Appendix).

Who gains from lower inflation? The policy functions presented above just show changes in consumption and savings due to lower inflation in the initial period in which the shock realizes. To further disclose how inflation affects different agents, we calculate the change in expected lifetime utility due to the permanent change in the inflation rate from 3% to -3%. Denote by $v_3(s, y)$ the expected lifetime utility of a household with income y and wealth s at time 0 for the case of an unchanged inflation rate of 3%. The expected lifetime utility at time 0 for the case of a permanent change in the inflation rate from 3% to -3% is given by $v_{-3}(s, y)$. The expected lifetime utilities are given by $v(s, y) = E_0 \sum_{t=0}^{\infty} \beta^t c_{i,t}^{1-\sigma}/(1-\sigma)$. The reduction in the inflation rate increases expected lifetime utility of a household in the initial state (s, y) if $v_{-3}(s, y) - v_3(s, a) > 0$, and vice versa. To quantify the welfare consequences of the change in the inflation rate for a household of type (s, y) we express the differences in units of consumption. Therefore, we calculate the percentage change in consumption in the stationary equilibrium with an inflation rate of 3%, in each date and state, for the household of type (s, y) to be indifferent between an inflation rate of 3% and a permanent reduction in the inflation rate to -3%. The percentage gain g of the inflation reduction is implicitly given by $v_3(s, y; g) = v_{-3}(s, y)$ with $v(s, y; g) = E_0 \sum_{t=0}^{\infty} \beta^t \left((1+g)c_{i,t}\right)^{1-\sigma}/(1-\sigma)$.[14]

The solid black lines in the first column of Figure 4 shows $g(s, y)$ for the different income states. Furthermore, the figure splits $g(s, y)$ into the contribution of the initial debt deflation effect (DDE, see dotted lines) and of $g(s, y)$ without the initial debt deflation effect ($w/o\ DDE$, see dashed line). The contribution of the latter effect is given by $\widetilde{g}(s, y)$, implicitly defined by $v_3(s, y; g) = v_{-3}(\tilde{s}, y)$ where $\tilde{s}$ is given by $\tilde{s}/0.97 = s/1.03$,[15] such that the initial debt deflation effect, which as been found to be dominant

[14] This implies $(1+g)^{1-\sigma} v_3(s, y) = v_{-3}(s, y)$ which yields $g(s, y) = \left(\frac{v_{-3}(s,y)}{v_3(s,y)}\right)^{\frac{1}{1-\sigma}} - 1$. If the household of type (s, y) prefers the economy with the lower inflation rate, i.e. $v_{-3}(s, y) - v_3(s, y) > 0$, $g(s, y)$ measures by how much, in consumption terms.

[15] Put differently, the effect $v_{-3}(\tilde{s}, y) - v_3(s, y)$ is the difference in expected lifetime utility between a household who lives in an economy with an inflation rate of 3% and has a real value of beginning of period wealth $s/1.03$ and another households who lives in an economy with a permanent reduction in the inflation rate to -3% and has a real value of beginning of period wealth $\tilde{s}/0.97 (= s/1.03)$.

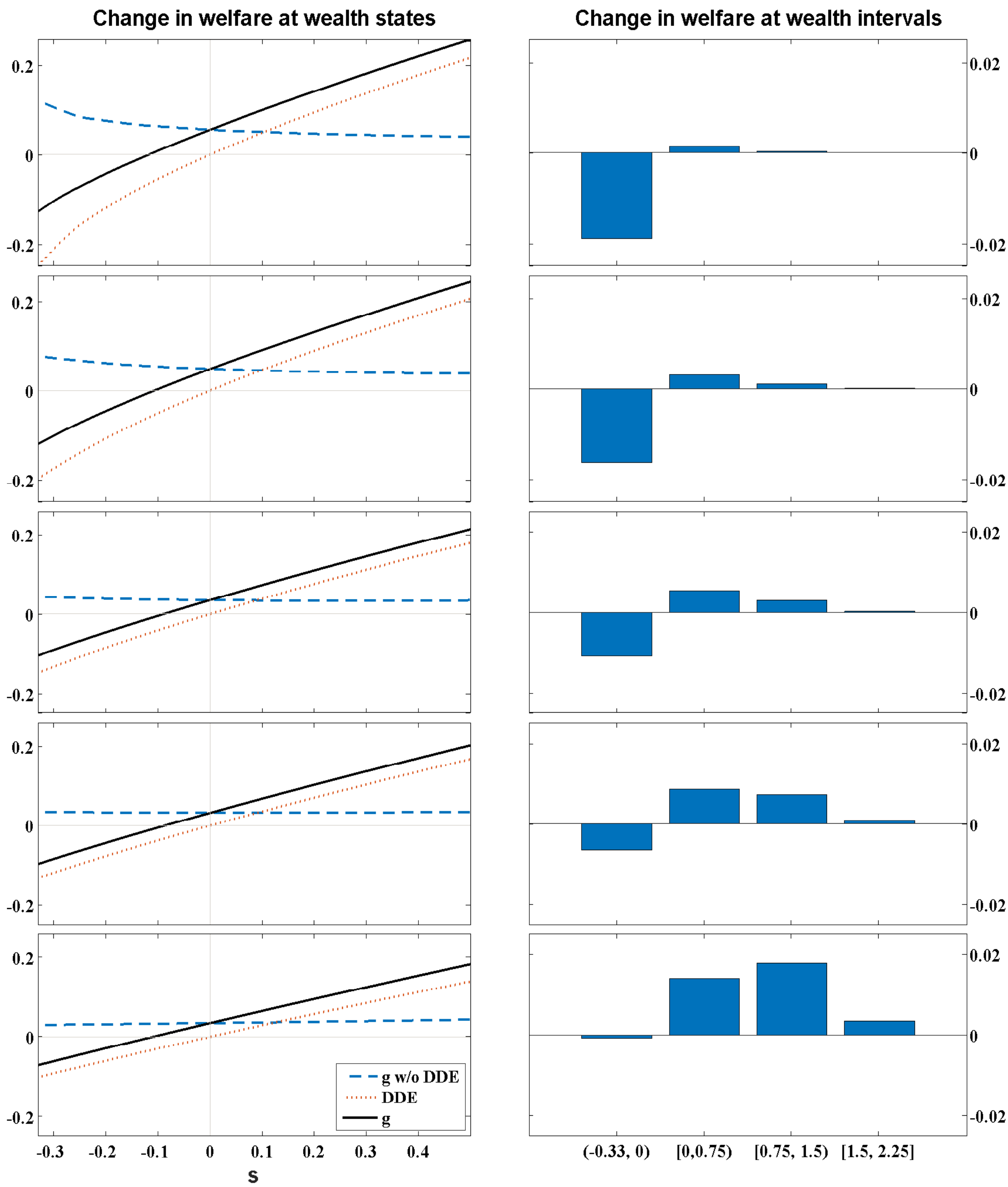

Figure 4: Welfare contributions of redistristributive effects for all income states [first row: y_1, second row: y_2, thirst row: y_3, forth row: y_4, fifth row: y_5]

in the model of Section 2, is shut down. The remaining effect $\widetilde{g}(s,y)$ is then given by the debt limit effect under a lower inflation rate as well as the debt deflation effects for $t \geq 1$,[16] while the contribution of the initial debt deflation effect is then simply the residual to $g(s,y)$.

Apparently, the welfare contribution of the initial debt deflation effect tends to be negative (positive) for households with initial debt (positive wealth) in all income states. The remaining effect $\widetilde{g}(s,y)$ tends to increase expected lifetime utility, in particular, of constrained borrowers and households with a high probability to be constrained in future periods by increasing the borrowing limit. However, the debt limit effect tends to increase expected lifetime utility also of wealthier agents due to the increase in the effective debt limit. The overall effect is that agents with relatively high initial debt (especially the constrained borrowers) suffer due to the dominating initial debt deflation effect. For these agents, the debt limit effect is dominated by the adverse effect on the initial debt repayment. Agents with positive wealth benefit from the reduction in the inflation rate due to both a higher real wealth in the initial period and higher borrowing limits in future periods in which they might be constrained. Importantly, agents with relatively low initial debt also benefit from the lower inflation rate, even though their initial debt repayment increases. This is due to the beneficial debt limit effect which allows to increase borrowing in future periods, where these agents might be constrained. In these cases, the debt limit effect dominates the initial debt deflation effect.

The second column of Figure 4 shows the welfare gains of agents with an initial net wealth from a common set of initial net wealth positions for each income level. Thus, it weights the welfare effects with the mass of agents from the same net wealth set. Apparently, welfare of agents with a particularly high debt position who suffer most from a reduction in the inflation rate contribute to a large (small) amount to the welfare of agents with a low (high) income. The mass of agents with intermediate debt or wealth levels and the cumulative welfare effects of these groups of agents increases with the income level. Notably, these agents hardly face a binding borrowing constraint. Yet,

[16] Notably, this also includes changes in the real interest rate induced by changes in the equilibrium amount borrowed (see also Figure 6).

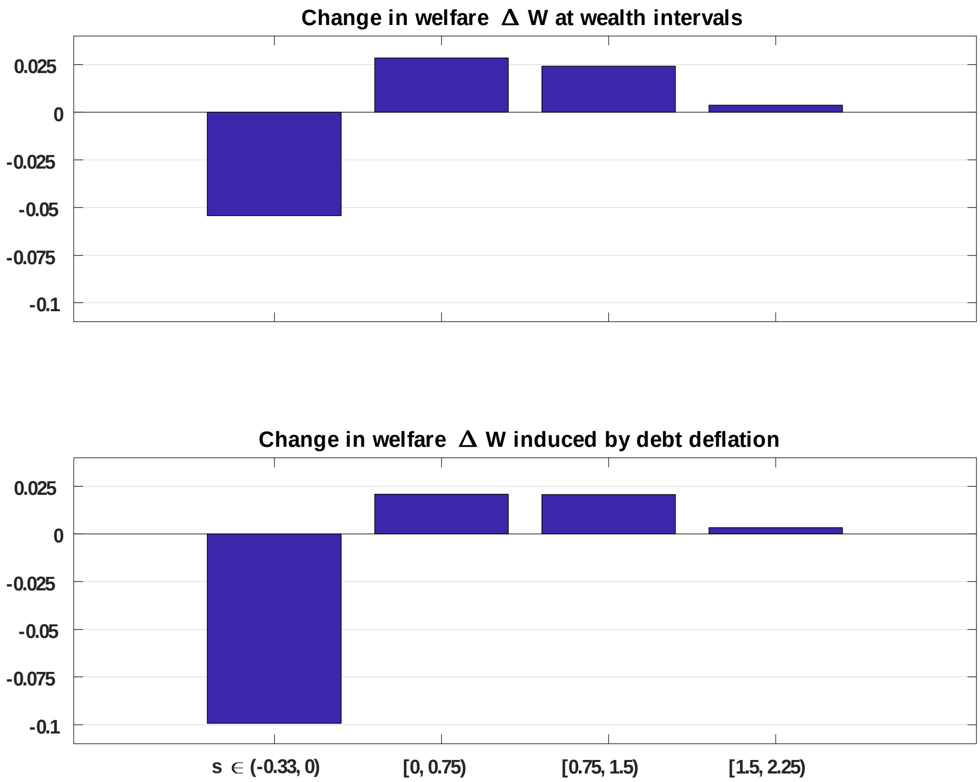

Figure 5: Welfare aggregated over net wealth for all income states (upper panel: total effect; lower panel: DDE)

they assign a positive probability of being constrained in the future such that a relaxation of the effective debt limit is beneficial for them. Thus, the positive debt limit effect of these agents can potentially outweigh the adverse debt deflation effect of highly indebted borrowers.

How is aggregate welfare affected by a reduction in the inflation rate? In the previous analysis, we have shown how individual agents' welfare is affected by a reduction in the inflation rate. The question we address now is how its overall effect is on social welfare. Computing this effect is straightforward: As defined above, $v_\pi(s,y)$ is the expected lifetime utility of household of type (s,y) for the inflation rate π and $g(s,y)$ measures by how much this household prefers the economy with an unexpected permanent change in the inflation rate from 3% to -3% in consumption terms, $g(s,y) = \left(\frac{v_{-3}(s,y)}{v_3(s,y)}\right)^{\frac{1}{1-\sigma}} - 1$.

The change in aggregate welfare is then given by $\Delta W = \left(\frac{\lambda_3(s,y)v_{-3}(s,y)}{\lambda_3(s,y)v_3(s,y)}\right)^{\frac{1}{1-\sigma}} - 1$, where λ_3 is the wealth distribution before inflation is changed. Given the parameter values from above, an unexpected reduction in the inflation rate from 3% to -3% increases welfare by $\Delta W = 0.0021\%$. Hence, the losses faced by highly indebted agents are dominated by the gains of the remaining agents. This can be seen from aggregating welfare over the different income states (see Figure 4), which is shown in Figure 5. The upper panel of this Figure shows that utilitarian welfare in fact increases by the reduction in inflation. This differs from the welfare effects of a pure debt deflation scenario (i.e. an exclusive change in real initial net wealth by -3%), which leads to an unambiguous reduction in social welfare (see lower panel of Figure 5).

Finally, we want to assess how the reduction of the inflation rate affect agents over time. When the inflation rate is reduced, the wealth distribution is initially consistent with an inflation rate of 3%. At the beginning, the adverse debt deflation effect tends to reduce agents' funds available for consumption, such that utilitarian social welfare decreases compared to the case of an unchanged inflation rate (of 3%). From then onwards, the economy converges to a new stationary wealth distribution induced by the lower inflation rate of -3%. Under the latter inflation rate, the effective debt limit is reduced, such that agents' access to external funds is less constrained. As shown above, the debt limit effect, is then positive for all agents such that social welfare is larger in the new stationary equilibrium. The upper panel of Figure 6 shows how aggregate (period) utility in consumption unit falls on impact and then increases to a higher level in the stationary equilibrium under the lower inflation rate. Given that borrowing increases with lower inflation, the real interest rate increases by 12 b.p. in the new steady state (see bottom panel of Figure 6). For the current policy experiment, the initial adverse debt deflation effect is thus dominated by the overall beneficial effect on the debt limit, whereas the opposite result has been found in the model with permanent borrowers and lenders (see Section 2).[17]

[17] It should be noted that for larger reductions in the inflation rate, which might violate the zero lower bound, the debt deflation effect can be dominant, such that social welfare decreases.

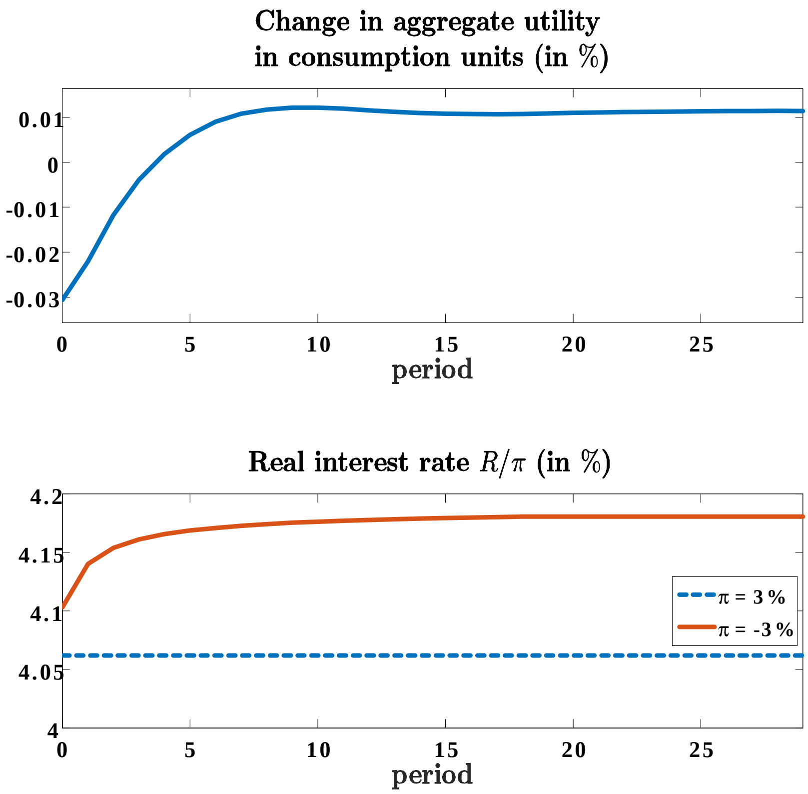

Figure 6: Paths of aggregate utility and the real interest rate

4 Conclusion

In this paper we analyze how financial frictions contribute to redistributive effects of monetary policy. We explore a novel mechanism of monetary non-neutrality, which is based on borrowing constraints related to current income. Such debt limits, for which broad empirical evidence exists, do not account for expected price changes until maturity, implying that monetary policy can alter the real terms of borrowing. A reduction in inflation tends to increase the maximum amount of debt that can be issued while it also raises the beginning-of-period stock of debt to be repaid. The impact of the inflation rate depends on the probability of borrowers to be unconstrained at maturity. The lower this probability is, the smaller is the beneficial effect of lower inflation for borrowers. The effect on the debt limit is opposite to a debt deflation effect when borrowers are initially indebted. The overall effect is therefore ex-ante ambiguous and depends on the initial debt/wealth position as sell as the willingness to borrow. If the probability for borrowers to be constrained in the future is relatively high, the debt limit effect dominates, such that a reduction in inflation can enhance social welfare. We show that lower inflation particularly benefits agents with low initial debt who borrow today or in the future by relaxing effective borrowing constraints, whereas highly indebted borrowers suffer from the adverse effect of debt deflation. For a calibrated model, we find that a reduction of inflation from 3% to -3% can nonetheless enhance (utilitarian) social welfare.

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A Appendix to section 3.2.2

A.1 Transition matrix

The transition matrix of idiosyncratic income with the conditional probabilities $Q(a_l|a_k)$ is given by

$$Q = \begin{pmatrix} 0.619 & 0.305 & 0.070 & 0.005 & 10^{-5} \\ 0.345 & 0.423 & 0.200 & 0.032 & 0.001 \\ 0.030 & 0.198 & 0.415 & 0.315 & 0.041 \\ 0.006 & 0.074 & 0.303 & 0.475 & 0.143 \\ 10^{-6} & 10^{-4} & 0.011 & 0.173 & 0.815 \end{pmatrix}$$

A.2 Calculation of the stationary equilibrium under a given inflation rate

We calculate the equilibrium interest rate, the decision rules and the time-invariant distribution under a given inflation rate π by using an endogenous grid point method. For this we proceed as follows:

I. Choose an initial value for R, i.e. R^0.

II. Calculate for R^0 the consumption policy function $c(s, a)$ with an endogenous grid point method (see Carroll, 2006) neglecting market clearing for loans.

III. Given the wealth policy function $s'(s, a)$, compute the implied stationary distribution $\lambda(s, a)$ (see below).

IV. Check market clearing for loans: $|\sum_{s,a} \lambda(s, a)s'(s, a)| < \epsilon$, where $\epsilon > 0$ is relatively small. If $|\sum_{s,a} \lambda(s, a)s'(s, a)| < \epsilon$, stop: R is the equilibrium nominal interest rate. If $|\sum_{s,a} \lambda(s, a)s'(s, a)| > \epsilon$ update the nominal interest rate and go back to step 2.

V. The interest rate is updated as follows. We start by calculating the policy functions for two different interest rates R^0 and R^1 with $R^0 < R^1$ and $\sum_{s,a} \lambda(s, a)s'(s, a) \leq 0$ for R^0 and $\sum_{s,a} \lambda(s, a)s'(s, a) \geq 0$ for R^1. New updates of the interest rate are then calculated by using bisection.

The endogenous grid point method for a given nominal interest rate is computed as follows:

1. Discretize next period wealth space $s' = \left\{s'_1, s'_2, ..., s'_{a_4}, ..., s'_{a_3}, ..., s'_{a_2}, ..., s'_{a_1}, ..., s'_m\right\}$, $s'_i < s'_{i+1}$ with $s'_1 = s'_{a_5} = -\gamma a_5$ and $s'_{a_i} = -\gamma a_i$. Thus, the discretized 2-dimensional state space is given by $\{s'_1, s'_2, ..., s'_m\} \times \{a'_1, a'_2, ..., a'_n\}$, where a'_k, $k = 1, ..., n$, are the possible income states. Choose a stopping rule parameter $\epsilon^{egm} > 0$.

2. Make a guess for the consumption policy function $c^0(s'_i, a'_k)$, where $k \in \{1, ..n\}$ and the guess is computed by $c^0(s'_i, a'_k) = -s'_i/R + s'_i/\pi + a'_k$, at all states in the discretized state space.

3. Update the consumption policy function (using two auxiliary functions $\hat{c}^0(s'_i, a_k)$ and $\hat{s}(s'_i, a_k)$):

   - Use $c^0(s'_i, a'_k)$ to compute a guess for current period consumption using $\hat{c}^0(s'_i, a_k)$ for future period wealth s'_i and some current period income a_k by using the Euler equation:

   $$\hat{c}^0(s'_i, a_k) = \left(\beta(R/\pi)\left(p_{k1}c^0(s'_i, a'_1)^{-\sigma} + p_{k2}c^0(s'_i, a'_2)^{-\sigma} + ... + p_{kn}c^0(s'_i, a'_n)^{-\sigma}\right)\right)^{-1/\sigma} \tag{25}$$

   where $s'_i \geq s'_{a_k}$ at today's income state a_k due to the borrowing constraint.

   - Use the budget constraint and the auxiliary function $\hat{c}^0(s'_i, a_k)$ to compute current period wealth $\hat{s}$ for s'_i and a_k:

   $$\hat{s}(s'_i, a_k) = \left(\hat{c}^0(s'_i, a_k) + s'_i/R - a_k\right)\pi \tag{26}$$

   - Calculate the updated consumption policy function at $(s'_i, a'_k) \in \{s'_1, s'_2, ..., s'_m\} \times \{a'_1, a'_2, ..., a'_n\}$ as follows:

     IF $s'_i \leq \hat{s}(s'', a'_k)$, for $s'' = -\gamma a'_k$, such that $\hat{s}(-\gamma a'_k, a'_k)$ is the beginning-of-period wealth in the future period of a household with income a'_k who borrows the maximum amount. This implies that a household with the same income a'_k but with beginning-of-period wealth s'_i that is smaller than

$\hat{s}(-\gamma a'_k, a'_k)$ is borrowing constrained as well. The updated consumption policy function at (s'_i, a'_k) is then computed by

$$c^1(s'_i, a'_k) = \gamma a'_k/R + s'_i/\pi + a'_k \tag{27}$$

and for end-of-period wealth given by

$$s'' = -\gamma a'_k. \tag{28}$$

ELSE IF $s'_i > \hat{s}(-\gamma a'_k, a'_k)$ the borrowing constraint is not binding at beginning-of-period wealth s'_i and income a'_k in the future period. The updated consumption policy function c^1 at (s'_i, a'_k) is then calculated using the implicit definition $\widetilde{c}^0(\hat{s}(s'_i, a_k), a_k) = \hat{c}^0(s'_i, a_k)$. Then, $c^1(s'_i, a'_k)$ is computed by a linear interpolation of $\widetilde{c}^0(\hat{s}, a)$ at (s', a'), where s' take on-grid values. The wealth policy function at (s'_i, a'_k) is then computed by using the budget constraint.

- IF $||c^1(s'_i, a'_k) - c^0(s'_i, a'_k)|| < \epsilon^{egm}(1 + ||c^0(s'_i, a'_k)||)$, stop.
  ELSE $c^0(s'_i, a'_k) = c^1(s'_i, a'_k)$ and start again step 3.

The stationary distribution for given policy functions is computed by calculating the normalized eigenvalue of the Markov transition matrix:

1. We add further wealth states to get a finer grid than the one used for the calculation of the policy functions (from 5 to 100 thousand grid points for s) and we calculate the wealth policy function values for the new states.

2. Calculate the transition probability of being in the state (s_j, a_l) in the next period if the current state is (s_i, a_k) and denote it by $P((s_i, a_k), (s_j, a_l))$. This probability is computed by $P((s_i, a_k), (s_j, a_l)) = Q(a_l|a_k) * I(s'(s_i, a_k) = s_j)$, where $I(s'(s_i, a_k) = s_j) = 1$ if $s'(s_i, a_k) = s_j$ and 0 otherwise. The Markov transition matrix is then given by the transition probabilities $P((s_i, a_k), (s_j, a_l))$ for all combinations of states.

3. Compute the eigenvector of the transition matrix associated with the largest eigenvalue (which is one). The stationary distribution on the grid is then given by the

normalization of this eigenvector.

A.3 Calculation of the transition path to the new stationary equilibrium

In period 0 the economy is in the stationary equilibrium under an inflation rate of 3%. In period 1 the inflation rate unexpectedly and permanently changes to -3%. The economy then leaves the old stationary equilibrium in period 1 and converges to the new stationary equilibrium under an inflation rate of -3%. The transition path is computed as follows (see e.g. Rios-Rull, 1999):

- Calculate the stationary equilibria for the two inflation rates of 3% and -3% as described above and denote the respective stationary distributions by $\Phi_{3\%}$ and $\Phi_{-3\%}$.
- The beginning of period distribution in period 0 is denoted Φ_0 and given by $\Phi_0 = \Phi_{3\%}$. The beginning of period distribution after the economy has converged into the new stationary equilibrium is denoted Φ_∞ and given by $\Phi_\infty = \Phi_{-3\%}$. Note that the beginning of period distribution in period 1 is the same as in period 0, i.e. $\Phi_1 = \Phi_0$, because the change in the inflation rate is not expected at time 0.
- Calculate the associated value function $v_0(s, a)$ in period 0 (which gives the expected lifetime utility of a household who is in the state (s,a) in period 0) and the value function in the new stationary equilibrium denoted by v_∞.
- Calculate the transition path:
  1. Assume that the transition into the new stationary equilibrium takes T periods. This implies $\Phi_T = \Phi_\infty$ and $v_T = v_\infty$.
  2. Guess a sequence of interest rates $\{\hat{R}_t\}_{t=1}^{T-1}$ and choose stopping rule parameters $\epsilon^s > 0$ and $\epsilon^\Phi > 0$.
  3. Since we know $v_T(s, a)$ and have a guess $\left\{\hat{R}_t\right\}_{t=1}^{T-1}$ we can solve for $\{\hat{v}_t, \hat{c}_t, \hat{s}_{t+1}\}_{t=1}^{T-1}$ backwards.

4. Use the policy functions $\{\hat{s}_{t+1}\}$ and $\Phi_1 = \Phi_0$ to iterate the distribution forward to get $\hat{\Phi}_t$ for t = 2,...,T.
5. Use $\left\{\hat{\Phi}_t\right\}_{t=1}^{T}$ to compute $\hat{A}_t = \int \hat{s}_{t+1} d\hat{\Phi}_t$ for t=1,...,T. Check for debt market clearance: If

$$max_{1\leq t<T} \left|\hat{A}_t\right| < \epsilon^s$$

go on. If not, adjust guesses for $\left\{\hat{R}_t\right\}_{t=1}^{T-1}$ and go back to step 3.
6. Check for $\left\|\hat{\Phi}_T - \Phi_T\right\| < \epsilon$. If yes, the transition converges smoothly into the new stationary equilibrium. If not, go back to step 1 and start again with a higher T.

- Note that the solution for v_1 is the value function at time 1 after the inflation rate has decreased. Thus, $v_1(s,a)$ is the expected lifetime utility of a household with income a and beginning of period 1 wealth s who has just been hit by the change in the inflation rate. This lifetime utility takes into account all the transition dynamics which the household is going to live through.

B Additional figures3.2.2

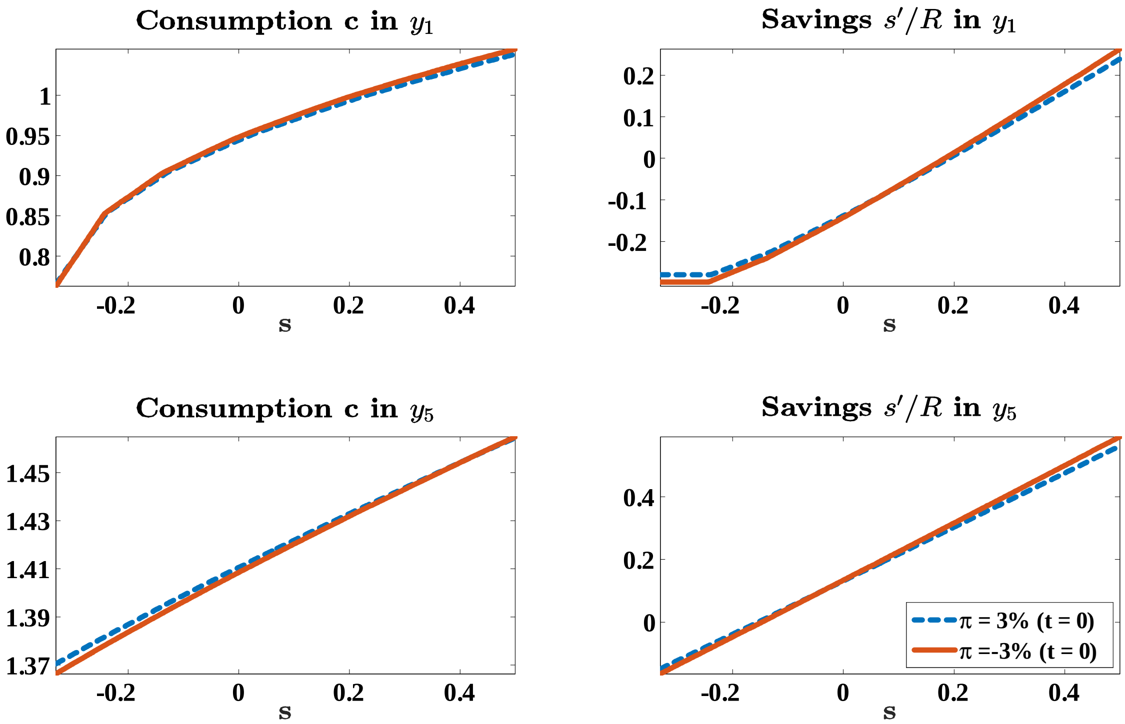

Figure 7: Policy functions for consumption and savings in period 0 over a larger state space

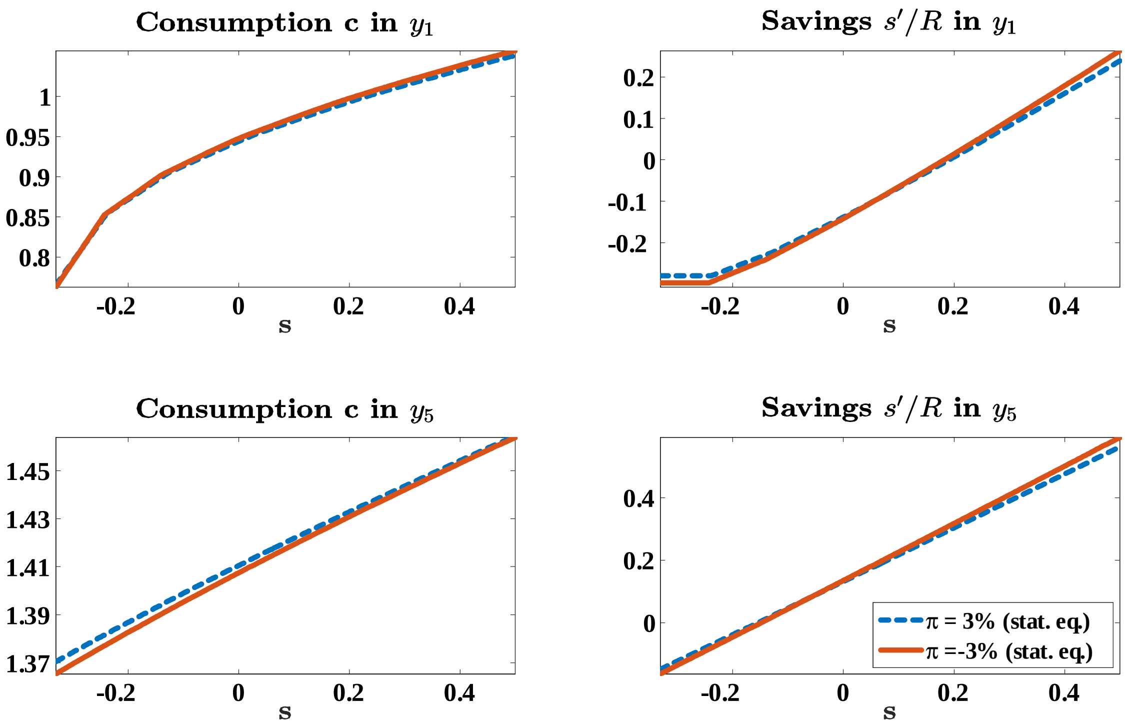

Figure 8: Policy functions for consumption and savings for stationary wealth distributions