Can Capitalists Remain Rich Without Working?

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Abstract. The present article builds on two hypotheses: 1) Productivity growth is uneven across sectors and 2) Capital, more precisely, durable produced factors of production, are produced in sectors with relatively high productivity growth. In an otherwise standard model of accumulation and technological progress it is shown that the share of capital in total income tends to zero in the long-run. This shift in factor-shares goes hand in hand with a sectorial shift: Employment and expenditure shares are shifted to sectors with low productivity growth. The conclusions about factor shares contradict the widely accepted stylized fact of a non-diminishing share of capital income in total income. However, a more disaggregated view on the evolution of factor shares and the evolution of sectorial shares shows that the conclusions are perfectly compatible with empirical facts. (*JEL* E25, O41)

1. Introduction

This paper analyses the impact of unbalanced sectorial productivity growth on the evolution of the factor-shares of an economy.

The evolution of the shares of capital incomes and labor incomes in national income, is the object of one of Kaldor's famous 'stylized facts'. This stylized fact claims the approximate long-run constancy of factor shares. It has so far done fairly well in empirical tests and is widely accepted as a desirable property of any descriptive long-run model of accumulation and technological progress. Almost all growth models of the literature are indeed constructed such as to be compatible with Kaldor's stylized fact.

In most of these models an *aggregate* output is produced with the aggregate inputs capital and labor and the productivity of labor in this aggregate activity can grow without bounds. The aggregation of all sectors to a single one, producing an imaginary compound commodity is an extreme abstraction. Once deviating from such extreme aggregation and partitioning the economy in more then one sector, one also has to deal with several *sectorial productivities.* It seems clear, that productivity growth in reality is not the same across sectors:

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While a professional haircut today takes about the same time as it did many hundred years ago (has its quality much risen?), a middle-class car of most brands in 1997 is produced with about half the labor-input than a middle-class car in 1992 and, at the same time, the quality of the car has much risen.

A waiter today needs about the same time as ever to carry a plate from the kitchen to the customer's table, while his colleague, the cook, gets much technical help and while his former colleague, the dishwasher, has completely disappeared.

While a whole orchestra playing for a handful of a selected listeners can, to a certain extent, be replaced by one tape playing for an audience of millions, a live performance, in the taste of many, remains without close substitute. And playing a Vivaldi suite for violin needs the same labor today as at the time of Vivaldi. So does the production of a good violin.

One hour of personal attention, even if professional, cannot be fully replaced by the most sophisticated modern equipment: Personal teaching in small classes, a lawyer's advice in an increasingly complex world, direct consultation in the health system, personal assistance for the old, child care, etc.. The growth of labor-productivity in some of these areas may be bounded by natural limits.

Clearly, productivity growth is not balanced across sectors. *Unbalanced sectorial productivity growth will be the first of the two basic hypotheses of the present paper.*

Among the commodities which are made cheaper by technological progress are final consumption commodities, like cars or televisions, as well as 'produced factors of production' like machines, trucks or computers. In a one-sector growth model, durable produced factors of production (capital), are of course produced in the only existing sector with growing productivity. In a model with more than one sector, we have to decide in which sector, or in which sectors, capital is produced. The above examples suggest that the commodities that are subject to limits to productivity growth typically are final consumption commodities rather than durable intermediate goods. *The second of the two main hypotheses of the present paper will be that capital is produced in sectors with high productivity growth.*

In the present paper we try to understand the consequences of these hypotheses (1) on the evolution of sectorial shares in total employment and in total income and (2) on the evolution of factor-shares. As we shall see, the consequences on sectorial shares reflect well conventional perception of sectorial shifts, while the consequences on the factor-shares, at least at a first glance, are at odds with conventional wisdom.

The consequences of the hypotheses on the evolution of sectorial share can be roughly summarized as follows: If the sectors with low productivity growth produce, among others, commodities that cannot be substituted in consumption with products of sectors with high productivity growth, then the share in total expenditures and in total employment of these sectors will continuously rise. Looking at the above examples of low productivity growth commodities, one may expect that most of the

low productivity growth sectors belong to the service sectors of national accountings. Everybody knows, that, apart from unbalanced productivity growth, the service sector's part in GNP and employment have in fact much grown in the last decades and are still growing in all industrial countries. Our simple analysis will show that the essential elements are the elasticities of substitution, both the elasticities of substitution between the outputs of different sectors as that between the inputs capital and labor within sectors.

The link between unbalanced productivity growth and the evolution of sectorial shares has already been highlighted in previous literature. In the nineteenth century to explain or predict the decline of employment in agriculture and more recently by Baumol [4] and Baumol, Blackman, and Wolff [5], [6], to explain the rise of the service sector. These papers also provide empirical evidence for the sectorial shifts. Our conclusions concerning the evolution of sectorial shares do not much differ from theirs. We make more explicit the role of the elasticities of substitution in consumption and in production, and show that the sectorial shift cannot be prevented by he accumulation of capital in the high productivity growth sectors. However, since [4], [5], and [6], do not model the process of capital accumulation, they do not describe the evolution of the stock of capital and of the capital share, that comes together with the evolution of sectorial shares. This evolution of factor shares is in the center of our present interest.

What are the consequences of the two hypotheses for the evolution of the factor-shares? What should we intuitively expect? When the cost of producing durable intermediate commodities decreases continuously, while that of producing some consumption commodities does not, or does to a lesser extent, one may wonder whether the factor incomes generated by durable intermediate commodities can remain substantial in terms of primary factors and in terms of those commodities that are subject to natural limits to growth. Can 'capitalists' remain rich (relative to workers) without working if the capital they own can be reproduced with less and less labor? Do capitalists benefit from the productivity growth as much as workers do? Can the capital share in total income remain substantial?[1] As we shall see in a simple model

[1]This paper is about the capital share, rather than about some imaginary capitalists' income. Therefore, the adequate question is that about capital's share, which is a question about the long-run *functional* income distribution. However, in the main text of the present paper we assume the simple Kaldorian framework in which workers do not save and owners of capital do not work. In this framework, the functional income distribution also is the personal income distribution. By the term 'capitalist' we mean a dynasty of individuals, initially owning some capital (for reasons not explored here), continuously consuming a fraction of their rental incomes and reinvesting the residual. The specification allows the distinction between capitalists and workers and simplifies the presentation, but is not essential for the analysis. In an appendix we show that the conclusions also hold in the "representative-immortal-consumer with perfect foresight"- framework. Also note that by our somewhat provocative title we do not intend to suggest that individuals with high capital

with unbalanced growth and capital accumulation that formalizes the two hypotheses, the share of capital incomes must indeed fall in the long-run. The rise of the low productivity growth sectors goes hand in hand with a decrease in the capital share. This seems to lead into a direct conflict with Kaldor's widely accepted stylized fact of constant factor-shares. The conflict is only an apparent one. Our conclusion that Kaldor's stylized fact cannot hold eternally does not exclude the possibility that capital share does not fall (or does even rise) over extended periods. To the contrary, the conclusion that eventually the capital share has to fall, not only follows from the formal analysis but also follows from an extrapolation of observed trends between and within sectors, using the classification of sectors suggested by the model.

The remainder of this paper is organized as follows. In Section 2 we briefly review a version of the standard one-sector model of growth and accumulation. In assuming that capital is produced one-to-one with final output, it indirectly reflects our second hypotheses: the cost of producing capital in terms of labor decreases continuously. The price of capital in terms of final output trivially remains one and the price of capital in terms of labor tends to zero at the rate of the technological progress. Nevertheless, capital incomes in terms of labor are asymptotically constant. In the standard neoclassical model capital incomes remain substantial despite the falling price of capital, simply because capital accumulates at the rate at which its price declines. Capitalists *can* remain rich, even without working. In the long-run capital incomes grow exactly at the rate of labor incomes.

We then observe that already by only endogenizing the supply of labor in the one sector model, the capital share deteriorates, if *total* incomes (including the value of leisure) rather than actual (or net) incomes are considered. While leisure is a 'commodity' which is subject to natural limits to growth in the purest way, it cannot be transferred from one person to another. In Section 3 we introduce transferable commodities that are subject to natural limits to growth, which we call direct services. Capital as well as some final consumption commodities are produced in an 'industrial sector', identical to the single sector of the standard model. We show that capital incomes in terms of labor, in terms of actual incomes, and in terms of direct services do deteriorate in the long-run. Capitalists can no longer retain their wealth without working. The capital share tends to zero.

In Section 3 the economy consists of only two sectors, the low productivity sector uses labor only, and the assumed utility functions are rather specific. In Section 4 we generalize these aspects and suggest a sectorial partition of the economy that is appropriate to reformulate the hypotheses in a multi-sector economy. The grouping of sectors is based on the elasticities of substitution in consumption of the commodities of different sectors and the elasticity of substitution in production of capital and labor.

incomes *do* indeed not work.

The hypotheses then lead to a number of conclusions about sectorial shares and factor shares. In Section 5 we provide some empirical support for these conclusions, using the International Sectorial Date Base of the OECD.

In Section 6 we discuss the robustness of the results with respect to some extensions: endogenous technological progress, quality growth, human capital, imperfect competition, existence of a third factor (land).

2. THE STANDARD MODEL

We first briefly review a version of the standard one-sector model. A homogenous output $F(A_tL, K)$ is produced with labor L and capital K, where F is a standard neoclassical production function (i.e. it is a linear homogeneous concave production function with strictly decreasing, strictly positive marginal products and it satisfies the Inada conditions, $\lim_{K\to\infty} F_K = \lim_{L\to\infty} F_L = 0, \lim_{K\to 0} F_K = \lim_{L\to 0} F_L = \infty$). Labor-augmenting progress raises the parameter A_t at the constant rate $\widehat{A} > 0$. The output can either be consumed or it can be 'saved' and transformed (one to one) into capital. Capital depreciates at a rate $\delta > 0$. For the sake of the exposition assume labor incomes are completely consumed and savings arise only from capital incomes. We consider the simple Kaldorian case in which a constant fraction s of rental incomes is saved. This describes a version of the standard model of accumulation with exogenous technological progress. As is well known, this model possesses a globally stable steady state, defined by the constancy of the ratio of capital to labor in efficiency units $k_t = \frac{K_t}{A_tL}$.

Since the production of capital is subject to the same productivity growth as the production of final output the price of capital in terms of final output trivially remains one and the price of capital in terms of labor tends to zero at the rate of the technological progress. Since in the steady state the marginal productivity of capital is constant, the rate of return to capital is constant too and the ratio of interest rate to wage tends to zero too. Nevertheless, in the stable steady state, capital incomes grow exactly like labor incomes. The reason of course is that capital accumulates at the rate of technological progress (since k_t and L are constant), that is, capital accumulates exactly at the rate at which its price and its return deteriorate. The ratio of capital incomes to labor incomes, $\frac{r_tK_t}{w_tL_t}$, and therefore the capital share, $\frac{r_tK_t}{w_tL_t+r_tK_t}$, are strictly positive constants in the steady state. Capitalists benefit from the productivity growth of labor exactly as workers and can sustain their wealth and consumption without ever working.

Flexible labor supply in the standard model. In Section 3 we introduce commodities that are subject to natural limits to growth and show that this leads to a diminishing share of capital incomes. Before we do so consider the following minor deviation from the standard setting that already allows to combine the idea of

declining relative wealth of capitalists with that of constant capital share in actual income. This is to endogenize labor supply in letting workers' supply of labor depend on wages, assuming that they enjoy leisure (In the standard model sketched above labor supply is fixed exogenously). For instance, assume that a worker has a utility function $\left((1-l)^{\rho}+(y)^{\rho}\right)^{(1/\rho)}$, where $(1-l)$ is his 'consumption' of leisure (with $l \leq 1$) and y his consumption of final output. Assume that in all other respects the model remains the standard model sketched above. Introducing labor into the utility function is a first step towards the introduction of the commodities that are subject to limits to growth. In fact, leisure, albeit not directly tradable, is the purest direct service, produced one to one with labor. Solving their choice problem given real wages w_t, workers reduce their labor supply with the increase of wages, if leisure is a complement to consumption, i.e. if $\rho < 0$. It is easy to show that, as before, in the long-run, wages w_t grow like labor productivity A_t, employment tends to zero, labor-income $w_t L_t$ (capital letters denote aggregate variables) and hence workers' consumption, tends to infinity, interest rates tend to a constant, capital and hence capital incomes tends to infinity at the same rate as actual wage incomes. Therefore, the *potential* or *full* income of workers $w_t N_L$ grows faster than capital incomes, in fact $\frac{w_t N_L}{r_t K_t}$ tends to infinity (where N_L is the total labor endowment, or the total number of workers). Considering leisure as a pure direct service that a person produces for himself and that he sells to himself at current wages, the share of capital incomes in full incomes tends to zero. Although the *net* or *actual* income ratio $(\frac{w_t L_t}{r_t K_t})$ tends to a strictly positive constant, capitalists become ever poorer in the sense that a worker has to work less and less to generate any capitalist's income. Of course, this is compatible with the stylized facts of constant factor shares, since the latter are expressed in actual and not in full income.

3. Deteriorating capital incomes

While leisure may be viewed as a direct service, it is not directly transferable. We will now introduce tradeable direct services to check whether this leads to a declining share of capital in actual (or net) incomes. Labor productivity in the production of one group of commodities rises exactly as in the standard model. Capital belongs to this group. Total output of such commodities is $F(A_t L_Y, K)$, where L_Y is the labor employed in this first sector (the industrial sector) and where F has the same properties as in the previous section. As does the standard model, we assume that A_t grows at the constant rate $\widehat{A}$. Labor-productivity in a second group of commodities (the direct service sector) cannot grow beyond some natural bounds. For simplicity we first assume that there is no technological progress in this sector and that no capital is needed. Later we generalize this, allowing for capital as a factor of production of direct services as well. The output in the direct service sector is the amount of labor L_D employed in that sector. As before we stick to the benchmark case in which

workers do not save and capitalists do not/cannot work.

At each moment workers choose leisure consumption of industrial commodities and of services to maximize their utility

$$[(1-l)^\rho + (y_{Lt})^\rho + (d_L)^\rho]^{\frac{1}{\rho}}$$

subject to the budget constraint $Y_{Lt} + w_t d_L - w_t l \leq 0$, where $l \leq 1$ is the amount of labor offered by one worker, Y_{Lt} and d_L are a worker's demand for industrial commodities and services, where w_t is the current wage as well as the price of direct services, and where $\rho < 1$.

A capitalist chooses consumption y_K and d_K of the two commodities to maximize his utility $[(y_K)^\rho + (d_K)^\rho]^{(1/\rho)}$ subject to the constraint $y_K + w_t d_K \leq (1-s) r_t (K_t/N_K)$, where r_t is the current interest rate, s is the exogenous saving rate, and where K_t is the current stock of capital, and where N_K is the number of capitalists. Each capitalist reinvests his savings $sr_t(K_t/N_K)$. Therefore, the rate of capital accumulation is $\widehat{K}_t = \frac{\dot{K}_t}{K_t} = sr_t - \delta$, where $\delta > 0$ is the rate of depreciation.

In what follows capital letters always refer to aggregate quantities, i.e. $L_Y = N_L l_Y, N_L l_D = L_D$, $N_L l = L$ are the total amounts of labor supplied to the industrial sector, to the service sector, and to both sectors, respectively, $N_L y_{Lt} = Y_{Lt}$, $N_K y_K = Y_{K,} Y = Y_{Lt} + L_K$ are the total demands for industrial output by workers, by capitalists, and by both groups, respectively, and $N_L d_L = D_L$, $N_K d_K = D_{K,} D = D_L + D_K$ are the total demands for services. The smaller the ratio N_K/N_L of capitalists to workers (given the aggregate stock of capital), the richer is each capitalist relative to a worker. However, all qualitative conclusions are independent of this ratio. As in all models with linear homogenous technologies the ratio $(K_t/A_t L_t) = k_t$ of capital to labor (in efficiency units) employed in the industrial sector plays a crucial role.

(1) We first show that $k_t = \frac{K_t}{A_t L_t}$ is bounded away from zero. Suppose to the contrary that k_t tends to zero. Then, because of the Inada conditions, the interest rate $r_t = \partial F(A_t L_t, K_t)/\partial K - \delta = f'(k_t) - \delta$ tends to ∞,where $f(k) = F(1,k)$. Hence the rate of capital accumulation $\widehat{K}_t = sr_t - \delta = sf'(k_t) - (1+s)\delta$ tends to infinity. Since $\widehat{K}_t = \widehat{K}_t - \widehat{A}_t - \widehat{L}_{Yt}$ and both $\widehat{A}_t = \widehat{A}$ and $\widehat{L}_Y^t$ are bounded $\widehat{K}_t$ must tend to infinity. This contradicts k_t tending to zero.

(2) Next, we show *that* $w_t \to \infty$. Suppose not, i.e. suppose that w_t remains bounded. At each moment of time wages equal the marginal product of labor, $w_t = \partial F(A_t L_t, K_t)/\partial L = A_t(f(k_t) - f'(k_t)k_t)$. Since w_t is bounded and since $A_t \to \infty$, $(f(k_t) - f'(k_t)k_t)$ must tend to zero. Because $f(\cdot)$ is strictly concave, this requires that k_t tends to zero, which contradicts (1). Therefore $w_t \to \infty$.

(3) Maximizing a workers utility subject to his budget constraint yields

$$d_t = \frac{1}{(w_t)^{\frac{\rho}{1-\rho}} + 2},$$

$$l_t = \frac{(w_t)^{\frac{\rho}{1-\rho}} + 1}{(w_t)^{\frac{\rho}{1-\rho}} + 2}, \text{ and}$$

$$y_{Lt} = \frac{1}{(w_t)^{\frac{\rho}{1-\rho}} + 2}(w_t)^{\frac{1}{1-\rho}}.$$

(4) What happens to workers' choices when wages rise?

Case $\rho < 0$: In the case of bad substitutes ($\rho < 0$), a worker's supply of labor as well as demand for direct services tend to $\frac{1}{2}$, and his demand for industrial output tends to infinity, when wages w_t tend to infinity. The amount he works to finance the purchase of industrial output, $\frac{y_{Lt}}{w_t} = \frac{1}{(w_t)^{\frac{\rho}{1-\rho}}+2}(w_t)^{\frac{\rho}{1-\rho}}$, tends to zero. In the limit he spends all his labor income on the purchase of direct services. He does not bother to give up much leisure or much consumption of direct services for additional consumption of the industrial output of which he already consumes so much.

Case $\rho > 0$: On the other hand, in the case of good substitutes ($\rho \in (0, 1)$), d_t tends to zero and l_t tends to one. The consumer now wants to substitute leisure and direct services with industrial output which becomes cheaper and cheaper in terms of labor and direct services.

Although, in reality many or even most direct services can be substituted by some specifically designed industrial commodities (hand-made shoes by industrial shoes, hand washing by machine washing), we gave examples for other direct services that can't. Those that *can* be substituted *will* in fact be substituted (this follows from (2) and $\rho > 0$). In the long-run they disappear. Only those that cannot be substituted by commodities that become ever cheaper, may survive. In a model with more than two sectors (see Section 4) the assumption will be that there exist services that cannot be substituted by industrial commodities. In the present section, with only one direct service, the interesting case for our issue is that of direct services that are bad substitutes to industrial commodities (in the other case the model converges to the standard one-sector model). In what follows we will assume that $\rho < 0$.

The assumption that leisure too is a complement to industrial output and direct services is not relevant for our analysis. It seems, however, to be the more realistic case (see [9]). Furthermore, it allows to work with a simple symmetric utility function. In the case that leisure is a complement to consumption, only the existence of commodities constraint by natural limits to growth prevent the supply of labor from tending to zero. While the supply of labor falls due to cheaper industrial output, it does not fall beyond a level necessary to finance the consumption of direct services.

(5) *The supply of labor to the industrial sector tends to zero.* Since at equilibrium total labor employed in the service sector equals total production of services we have that $D_{Lt} = L_{Yt}$. Thus

$$\lim_{t\to\infty} D_{Lt} \leq \lim_{t\to\infty}(D_{Lt} + D_{Kt}) = \lim_{t\to\infty} D_t = \lim_{t\to\infty} L_{Dt} \leq \lim_{t\to\infty}(L_{Dt} + L_{Yt}) = \lim_{t\to\infty} L_t.$$

For the case of bad substitutes (we have assumed $\rho < 0$) we have seen in (4) that $\lim_{t\to\infty} L_t = \lim_{t\to\infty} D_{Lt}$. It follows that

$$\lim_{t\to\infty} D_t = \lim_{t\to\infty}(D_{Lt} + D_{Kt}) = \lim_{t\to\infty} D_t = \lim_{t\to\infty} L_{Dt} = \lim_{t\to\infty}(L_{Dt} + L_{Yt}) = \lim_{t\to\infty} L_t,$$

and therefore that $\lim_{t\to\infty} D_{Kt} = 0$ and $\lim_{t\to\infty} L_{Yt} = 0$. Less and less labor is supplied to the industrial sector.

(6) We next observe that *industrial labor income grows without bound.* The income from industrial labor is $w_t L_{Yt} = w_t(L_t - L_{Dt})$. Because of (2) and (3) we have $\lim w_t L_{Yt} = \lim_t w_t(L_t - D_{Lt}) = \lim_t w_t \frac{(w_t)^{\frac{\rho}{1-\rho}}}{(w_t)^{\frac{\rho}{1-\rho}}+2} = \lim_t \frac{(w_t)^{\frac{1}{1-\rho}}}{(w_t)^{\frac{\rho}{1-\rho}}+2} = \infty$, since $\rho < 0$. The amount of labor supplied to the industrial sector tends to zero when wages rise. However, labor supply falls slower than wages raise, so that industrial labor income (in terms of industrial output) grows without bound.

We now want to show that from (6) it also follows that $A_t L_t$ tends to infinity. This would be obvious if k_t were bounded (see (8)). However, while we have shown that k_t is bounded away from zero we have not shown that it is bounded. In fact, if L_t falls at the rate at which A^t rises, we are in a case analogous to the neoclassical case without technical progress and with fixed labor supply. In the case of no depreciation k_t may then tend to infinity. However, even then, $\widehat{K}_t$ could to zero.

(7) We show that k_t is bounded. Suppose not, i.e. suppose that $k_t \to \infty$. Then $0 \leq \lim \widehat{K}_t = \lim[sf'(k_t) - (1+s)\delta] - \lim(\widehat{A_t L_{Yt}}) < -\lim(\widehat{A_t L_{Yt}})$. Hence $\lim \widehat{K}_t < 0$ and $\lim(\widehat{A_t L_{Yt}}) < 0$. Therefore, $\lim \widehat{Y}^t < 0$ and $\lim Y_t = 0$. This contradicts $y_{Lt} \to \infty$, which follows from (4). Thus k_t must be bounded.

Note that if there is no depreciation, then we can only show that $\lim \widehat{K}_t = 0$.

(8) We show that $A_t L_{Yt} \to \infty$. $A_t L_{Yt} = (w_t L_{Yt})\frac{A_t}{w_t}$. Hence, because of (6) it suffices to show that $\frac{w_t}{A_t} = (f(k_t) - f'(k_t)k_t)$ is bounded. This is the case if k_t is bounded. Thus the claim follows from (7).

Note that without depreciation the claim follows as well. One has then to use the fact that $\lim \widehat{K}_t = 0$ and to show in addition that $A_t L_{Yt} \to \infty$ if k_t tends to infinity (to see this one has to show that otherwise the supply of industrial output is bounded, which contradicts (4)).

(9) What happens to capitalists in the long-run?

(9a) *Capital income in terms of industrial output tend to infinity.* Since $k_t = \frac{K_t}{A_t L_{Yt}}$ does not tend to zero (Step (1)) it follows from (8) that K_t tends to infinity and then from (7) that $r_t K_t$ tends to infinity. Thus, capitalists become infinitely rich in terms of the industrial output.

(9b) *Capital income in terms of direct services tends to zero.* What about the evolution of their wealth in terms of direct services, i.e. what happens to $\frac{r_t K_t}{w_t}$? Capitalists' income in terms of direct services is $\frac{r_t K_t}{w_t} = \frac{(f'(K_t)-\delta)K_t}{A_t(f(K_t)-k_t f'(k_t))}$. The term

$\frac{f'(k_t)-\delta}{(f(k_t)-k_t f'(k_t))}$ is bounded and $\frac{K_t}{A_t} = k_t L_{Yt}$ tends to zero because of (5) and (7). It follows that $\frac{r_t K_t}{w_t}$ tends to zero too. The capital income in terms of direct services tends to zero. Consequently, and in accordance with (4), capitalists' demand for direct services D_{Kt} tends to zero too.

(9c) *The ratio of capital incomes to total labor incomes* $\frac{r_t K_t}{w_t L_t}$ *tends to zero.* This follows, since the ratio of capital incomes to industrial labor incomes $\frac{r_t K_t}{w_t L_t}$ is bounded and $\frac{L_{Yt}}{L_t}$ tends to zero.

4. A More General Version

So far we have assumed that are only two sectors. The two commodities were good substitutes and no capital was used in the LPG-sector. In reality, there can be LPG-sectors producing outputs that can be substituted by HPG-commodities, and, at the same time, others that can't. In addition in some of these LPG-sectors capital may be a bad substitute for labor, while in others capital may easily substitute labor. To get a more complete picture we therefore need a model in which different types of LPG-sectors coexist.

In this section we first introduce capital as a factor of production in the direct service sector of the two sector model. We then introduce a general assumption concerning the elasticity of substitution of consumers preferences, which replaces the specific utility function of the previous section, before we restate the hypotheses and their consequences in a multi-sector framework. In this framework, some low productivity growth (LPG) sectors may die out because they can be replaced in consumption by industrial products; some produce more output without gaining in their shares of total income or employment, because they can substitute labor by cheap capital; some sectors cannot substitute much labor by capital and grow in output due to rising shares in total employment; some sectors shift their activity from market activity to non-market activity.

4.1. Capital in the service sector. In the previous section we have assumed that direct services are produced with labor only. This assumption is not essential. The barber may use electric shavers and the professor may use overhead projectors and computers. For our conclusions to remain valid however, it is important however, that these instruments cannot replace their labor. Otherwise remaining in the framework of the previous section, assume that direct services $D = D(L_D, K_D) = [(L_D)^\gamma + (K_D)^\gamma]^{\frac{1}{\gamma}}$ are produced with labor L_D and capital K_D. We want to show that if capital and labor are complements in the production of services ($\gamma < 0$), then the share of capital, $\frac{r_t K_t}{w_t L_t}$, tends to zero when the labor-productivity in the industrial sector tends to infinity.

(1) As before one can show that $k_{Yt} = \frac{K_{Yt}}{A_t L_{Yt}}$ is bounded and bounded away from zero, that workers' total supply of labor is bounded away from zero, and that workers

will not much work for industrial output that is extremely cheap, i.e. that L_{Yt} tends to zero. Therefore, $\frac{r_t K_{Yt}}{w_t}$ tends to zero as before (if $F(\cdot)$ is a CES function with parameter α we have $\frac{r_t K_{Yt}}{w_t} = \frac{r_t K_{Yt}}{w_t L_{Yt}} L_{Yt} = (k_{Yt})^{\alpha} L_{Yt}$).

(2) Since k_{Yt} is bounded away from zero, $\frac{w_t}{r_t}$ tends to infinity with A_t (if $F(\cdot)$ is a CES function with parameter α, then$\frac{w_t}{r_t} = (k_{Yt})^{1-\alpha} A_t$). *Therefore the capital/labor ratio in the service sector* $k_{Dt} = \frac{K_{Dt}}{L_{Dt}}$ *tends to infinity.* It follows that $\frac{r_t K_{Dt}}{w_t L_{Dt}} = (k_{Dt})^{\gamma}$ tends to zero if $\gamma < 0$.

(3) Since $\frac{r_t K_t}{w_t}$ and $\frac{r_t K_{Dt}}{w_t L_{Dt}}$ both tend to zero and L_{Dt} does not tend to zero capital's share in total income tends to zero, $\frac{r_t K_t}{w_t L_t}$ tends to zero, and $\frac{r_t K_t}{w_t}$ tends to zero.

Thus, if there is only one LPG-sector, here the sector of direct services, then our conclusions remain valid if capital and labor are complements in this sector. If capital is a good substitute for labor in a service sector more and more labor will be replaced by capital in the production process of this service. Such a sector will not contribute to the deterioration of capital incomes or to the decline of employment in the industrial sector. On the other hand they do not change the conclusions as long as there exists at least one sector using capital as bad substitute for labor and producing outputs that are bad substitutes for all HPG-sectors and for those LPG-sectors that use capital and labor as good substitutes. Thus in a multi-sector model the first hypotheses will assume that their must be at least one LPG-sector of the right type.

4.2. General utility functions. We just saw that the elasticity if substitution between labor and capital in the production of LPG-commodities is important. In the previous section we have already seen that the elasticity of substitution in consumption between LPG-commodities and HPG-commodities plays an important role.

We generalize the definition of good or bad substitutes:

We say that *commodities h is (asymptotically) a bad substitute for commodity h' if* $\frac{u_h}{u_{h'}} \frac{y_h}{y_{h'}}$ *tends to zero when* $\frac{y_h}{y_{h'}}$ *approaches infinity,* where $\frac{u_h}{u_{h'}}$ are a worker's marginal rates of substitution between the two commodities. Otherwise we say that commodity h is (asymptotically) a good substitute for commodity h'. The conclusions about sectorial shares in employment and income as well as about factor shares of the previous section remain valid if rather then assuming that the elasticity of substitution in consumption between the industrial good and the direct service is constant and smaller than 1, it is assumed that the industrial good is asymptotically a bad substitute for direct services.

While the asymptotic complementariness of industrial commodities and services is decisive, the conclusions do not depend on the assumption that leisure is a complement to produced commodities. If labor supply is fixed exogenously for instance, then asymptotically, the full amount of labor income will be spent for services as

before, and the amount of labor supplied to the industrial sector tends to zero.

4.3. Growth in the direct service sector. While some examples of commodities subject to natural limits to growth have been given, there are few examples in which these limits are as strict as assumed (no productivity growth). The relevant feature in concerning the productivity bias is that the rate of the productivity growth in the industrial sector ($\widehat{A}$) is persistently strictly higher than the rate of productivity growth in the direct service sector, i.e. that productivity growth is persistently biassed across sectors. It is easy to check that the conclusions remain the same if $\widehat{A}$ is the difference of the labor productivity of the two sectors in the previous section.

4.4. You-can-do-it-yourself services. Suppose that some direct services consumed by a worker can be directly produced by himself, so that (as for leisure) no market transaction appearing in national income statistics needs to be involved, but that these services are nevertheless transferable from one person to another person (unlike leisure) and *can* be bought by others ('you can do it yourself' variety of direct services). As for direct services in the previous section, the share of these services in *total* income increases. However, assuming that any market transaction involves some cost, workers will prefer to perform the activity themselves, rather than to buy it. Thus, workers' share in the consumption of these activities are not reported in national accounting and the rise of their share in total hours worked and in total income does not directly effect the evolution of capital's share in reported income. In contrast, capitalists' decreasing demand corresponds to a decrease in market activity. In fact while almost every European bourgeoise household employed at least one servant 200 years ago, only the very richest can afford full time servants nowadays. The profession disappears not only because capital goods that are good substitutes become cheaper, but also because servants become unattainable for those that previously employed plentiful. Part of the work that was previously performed by servants, still is performed today, but without market-transactions being involved. Another example is the rising number of self-service restaurants. Waiting as a profession declines much faster than as an activity. Such 'You-can-do-it-yourself-services' weaken the directly reported consequences of our hypotheses as far as national accounts serve as the basis of observation.

4.5. Multi-sector economy. We now reformulate the hypotheses and restate the consequences in a multi-sector economy. There are H sectors, or compound commodities. Consumers preferences are defined for vectors of such commodities.

We first group the sectors of an economy in an adequate way. Groups are defined with respect to substitution properties and productivity growth.

Concerning the latter we partition the economy in only two classes: high productivity growth (HPG) sectors and low productivity growth (LPG) sectors. While it

is obvious that the productivity growth is high in some sectors and low in others, there need not be a clear cut dividing line between low and high productivity growth sectors in reality. Drawing this line in reality will remain somewhat ad hoc.

For the substitution of different commodities in consumption we use the above definition of (asymptotically) bad substitutes. For the substitution of capital and labor we stick for simplicity to CES production functions in the LPG-sectors and say that capital and labor are good substitutes if the elasticity of substitution is at least one (including Cobb-Douglas production functions) and that they are bad substitutes otherwise.

We partition the economy into the following groups:

Group 1 All HPG sectors.

Group 2 LPG sectors in which capital and labor are *good* substitutes

Group 3 LPG sectors in which capital and labor are *bad* substitutes and which produce outputs for which outputs of Groups 1 or 2 are (asymptotically) *good* substitutes.

Group 4 LPG sectors in which capital and labor are *bad* substitutes and which produce outputs for which outputs of Groups 1 or 2 are (asymptotically) *bad* substitutes.

The hypotheses now become: *Groups 1 and 4 are non-empty. Not all commodities in Group 4 are of the 'you-can-do-it-yourself' variety. Capital (durable produced factors of production) are not produced in Group 4.*

Group 4 may further be divided into a subgroup of 'you-can-do-it yourself' services (Group 4a) and a subgroup of commodities that are not of this variety (Group 4b). It is Group 4b that ought to be non-empty.

Taking into account all conclusions of the previous section and subsections, the consequences can be summarized as follows:

Group 1 HPG, output grows, share in employment and share in national income fall.

Group 2 LPG, output grows, shares in employment and share in national income fall.

Group 3 LPG, shares in employment and share in national income fall.

Group 4 LPG, output grows, share in employment and share in national income grow, capital share in Group 4's expenditures falls.

Since Group 4's share (Group 4b'share) increases (tends to one) it follows, that *in the long-run the capital share in national income falls and must tend to zero.* However, it also is obvious that this effect needs to arise only after Group 4's share has risen sufficiently.

5. SOME EMPIRICAL SUPPORT

The International Sectorial Data Base (ISDB) of the OECD [1] provides sectorial data that allow in principle to directly calculate sectorial productivity[2] growth, sectorial shares in total income and sectorial factor shares for all OECD member states. The ISDB contains information for about 20 different sectors in all OECD member states.

1)We have first checked on a purely descriptive basis whether our predictions are compatible with this data. We have done this for Canada, France, Greater Britain, and the USA, for which the corresponding data are sufficiently complete for the years 1970 to 1994.

In each country we have first calculated the average annual labor productivity rates over the years 1972 to 1994. We have then partitioned the sectors into a HPG-group and a LPG-group. Drawing the dividing between the two groups not too far away from average PG, this classification is the same in all considered countries. While average productivity growth in the (aggregated) service sector was less than in the industrial sector in most countries, the service sector contains some very progressive subsectors. Productivity growth in the service sectors 'Electricity, gas and water (EGW)' and 'Transport, storage and communication (TRS)' are much higher than average productivity growth in all industrial countries.

First, the average annual growth rate of the share in total income of LPG-groups for the years 1972 to 1994 was positive in all countries (0.8% in Canada, 1% in France, 1% in Greater Britain, 0,6% in the USA). Second, the average annual rate of change of the labor share in the LPG-group too has been positive in all countries (0,7% in Canada, 0,1% in France, 0,43% in Greater Britain, 0,48% in the USA).

Thus the share in national income (and employment) of the LPG-sectors have indeed grown and *the capital share in these sectors has declined.* Taking into account that the share in national income of these sectors, while growing, is still far from one, and that the capital share in other sectors have been increasing, the fact that the capital share in national income has so far not risen is perfectly compatible with the prediction that it has to fall in the long run. *If all the trends within sectors and between sectors continue, then the capital share must in fact eventually fall.*

Our analysis suggest that the two tendencies (increasing share of the LPG-groups, increasing share of labor in these groups) are accentuated if a finer sectorial partition were used. For instance, the sector 'Financing, Insurance, Real Estate and Business

[2]Note that the definition of 'productivity' used in the preliminary empirical check is not the same as that used in the previous sections. The former is the simply output per hour worked.

Services (FNI)' which in the aggregate is a LPG-sector contains as a subsector 'Machinery and equipment rental and leasing', the sector Community, social and personal services (SOC)' contains as a subsector 'public defense' or the subsector 'Laundries, laundry services'. A finer sectorial partition may shift these and some other subsectors from the LPG-group to the HPG-group or to the LPG-sectors that uses capital and labor as complements (Group 2 of the previous section). Since our analysis makes no prediction about the evolution of factor shares of such sectors, correctly classifying them as HPG-sectors would generally improve the prediction.

Note that all service sectors of the ISDB, except EGW and TRS, belong to the LPG-group in all countries. The fact that observed PG is very high in the sectors EGW and TRS is possibly partly due to increasing returns to scale or positive network externalities. The share of the sector TRS has declined in all countries except Greater Britain (the average annual growth rates are -0,2% in Canada, -0,18% in France, +/- 0% in Greater Britain, -0,3% in the USA). This is compatible with the predictions. In contrast, the value share of the sector EGW has much risen in all countries except Greater Britain (1% in Canada, 2% in France, -1% in Greater Britain, 0,9% in the USA). This contradicts the prediction that the value share of HPG sectors should fall. Possibly this is due to the fact, that assuming relative satiability in consumption of energy (by households or as intermediate input) as compared to the consumption of LPG-commodities is not very realistic, at least not in the short run. Since in all countries except Greater Britain, the labor share in the sector EGW has fallen, the tendency in this sector contributes to an overall fall in labors share.

2) An adequate empirical test of our conclusions would require testing a general equilibrium model providing estimates for elasticities of substitution in consumption between sectors as well as for productivity growth and elasticities of substitution in the production functions of each sector. However, already information about sectorial elasticities of substitution in production would allow for a regrouping of sectors that comes closer to the grouping of sectors in the previous section. Estimates for sectorial production functions and productivity growth for the US and France based on the above mentioned OECD sectorial data base are provided in [3]. In accordance with the sectorial partition of the previous section we take only LPG sectors with high elasticity of substitution (less than 2% average annual productivity growth[3] and with an estimated elasticity of substitution larger than one[4]). In this group of sectors the average annual growth rate of the share in total income are +0.8% in the US and +0.75% in France. The average annual growth rate of the labor share in the group

[3]The anual rate of productivity growth in [3] is the anual rate of reduction minimal costs at given prices.

[4]The estimates for the elasticities of substitution in all sectors are either larger than one in all years or smaller than one in all years. They are larger than one in the same two sectors in both countries.

of sectors' total expenditures was +0.62% in the US and +0.30% in France. Thus these estimates too, confirm the direction of the theoretical prediction. In the US (but not in France) both trends are even confirmed in each of the 7 sectors of the corresponding group.

6. Extensions and Limitations

The analyzed was extremely simple. In this section we consider some possible extensions.

6.1. Endogenous technological change.

Endogenous direction of change. The postulated growth bias is a *natural* bias. The productivity in the production process of direct services is bounded by technical or logical constraints. Since all the results depend on the bias in the direction of change, we may want to explain endogenously the direction change. In this case we may expect that more and more resources spent on R&D flow the LPG sectors, since the relative prices of these commodities increase and the relative profits from successful innovations, increasing productivity by a given amount, would increase over time. If there are technological asymmetries between innovation possibilities, even exploring all possibilities of productivity growth cannot unconditionally balance growth across sectors. The present paper exclusively deals with such residual and exogenous biases. In some classical LPG activities technological progress is not per se impossible. Labor becoming increasingly expensive, a fully automatized laser-haircutting machine with many styling options may one day make professional hand-hairdressing a luxury commodity or completely replace hairdressers. But education, curing, advocating, consulting will probably always require direct human engagement.

Endogenous rate of change. One may also wonder whether endogenizing the *rate* of technological progress would much change the conclusions. The profits of an innovation that decreases at a given rate the prices of HPG commodities in terms of wages will decrease over time. If research is mainly performed by labor (or human capital weighted labor), then one has to make sure that the productivity of research, i.e. the productivity increase induced by one hour of research grows at least the rate of productivity growth. This also has to be assumed in standard models of endogenous growth ([2], [8], [10]). In most of this literature successful innovators realize some monopoly profits, the expectation of which justified the innovative effort. If growth is to be sustained, these profits have to be sustained and their share will typically not tend to zero. This does not cause a conflict with our conclusions. The 'profits' to innovation in fact are 'quasi rents'. If there is free entry to innovative activity the expected profits from innovations will be fully paid to the resources that are necessary to innovate. If R&D uses capital as a bad substitute for labor or human capital, then

the expected profits from innovation will be completely translated into labor incomes in the long-run. Note that this depends on the assumption of free entry to R&D.

6.2. Quality growth. In our simple model we have assumed that all technological progress occurs in the form of productivity growth. It is apparent that in reality there has been a tremendous quality growth in many sectors.

Quality growth in HPG-sectors. Our arguments about the rising share of one part of the LPG sectors depend on the assumption that consumers will become relatively satiated in the HPG commodities of which they will have abundantly compared with the LPG-commodities of which they have relatively little (good asymptotic substitutes). This is a plausible assumption if it really is the *quantity* of the HPG good which becomes always cheaper. If it only is a *quality adjusted* quantity, then matters may be very different. It may be realistic to assume that people are satiable in quantity (relative to a scarce commodity, asymptotic bad substitute), while they are not satiable in quality. In this case our conclusions require that not all of the quality adjusted quantity growth is due to quality growth. The data of the OECD ISDB that we have used in the previous section do not allow to distinguish between productivity growth and quality growth. Possibly, the grouping in HPG and LPG sectors would be very different if this distinction could be made (i.e. if the grouping would be based on true PG alone).

Quality differentiation in the LPG-sectors. A second important omission may be that of *horizontal* product innovation in the LPG sector, in particular if one has in mind that most LPG sectors are service sectors. The growth of the service sector in reality goes hand in hand with the creation of many new types of services. To the extent that this increases the quality of capital used in the production of services (like razors for hairdressers or computers for lawyers and teachers) this can be easily covered by a minor variation of the model. To draw a more satisfying picture of reality we should allow for quality differentiation. For the conclusions about sectorial shares and factor-shares to survive the introduction of quality growth, one has to assume that consumers' relative satiation in HPG-commodities as compared to LPG-commodities is not obstructed by the presence of quality growth.

While including vertical and horizontally quality differentiation would certainly make the analysis more realistic, it would at the same time move us even further away from the data as measured in national accounting. In any data base which uses a constant number of sectors, horizontal product differentiation must eventually be translated into output variation in some sectors.

6.3. Human capital. The introduction of human capital would necessitate an adoption of the two hypotheses. First, the hypotheses of unbalanced productivity growth has to be extended: increasing the level of education or the level of skill of

an individual should not too much raise the productivity of an individual in some of the LPG sectors. Second, the hypotheses specifying where capital is produced has to be extended: human capital should be produced in LPG sectors (more precisely in Group 4 of Section 4).

6.4. Imperfect competition. Usually the term 'capital income' is not reserved to pure factor incomes. In practice, the joint ownership of a firm and of the capital the firm employs make it difficult to distinguish pure factor incomes to capital from pure profits. In the framework of the previous sections pure profits could not arise, since we assumed perfect competition with linear homogenous aggregate technologies (which is equivalent to free entry for small firms). If instead, imperfect competition plays a persistent role in the economy, then profits too will persist and there is no reason for expecting that unbalanced growth leads to declining profits, in particular when this profits arise in LPG-sectors. The share of the pure factor incomes to capital will still tend to zero.

6.5. Land. The notion of aggregate capital underlying the stylized fact of constant capital share not only includes the share to pure profits, but also does it not distinguish between reproducible capital, like machines, and non-reproducible capital, like land. For our issue the distinction is crucial. In fact, in many respects, the primary factor land is more like the primary factor labor than like the intermediate factor capital. As is time, space is not only an input for production, it is also directly consumed. Like time not spent at work, space not used up by machines is of great direct value to consumers. And like time, space cannot be completely replaced by any produced commodity. Furthermore, as long as no new planets are made habitable, space is the scarce resource par excellence (labor is only one if there is no population growth).

First, consider the standard framework of Section 2 with fixed supply of labor and land. Suppose that industrial output $F(A_tL, A_tE, K)$ is now produced with labor L, land E, and capital K.[5] Then, on a balanced growth path, land incomes and labor incomes grow at the same rate. The shares of the three factors are constant.

Next, assume that production of final output needs land and that consumers derive utility directly from the use of land, i.e. introduce land into the utility functions of workers, capitalists/land-lords $((1-l)^\rho + e^\rho + y^\rho)^{(1/\rho)}$, $(e^\rho + y^\rho)^{(1/\rho)}$ in the same way as industrial output or leisure. One can now verify that the share of land in total

[5] We assume that the productivity of land rises as that of labor. In fact, the arguments in the literature on induced innovation explaining why progress must be labor-augmenting and not capital-augmenting also lead to land-augmenting progress. Assuming symmetric innovation possibilities for land and labor leads to symmetric factor augmentation for land and labor (see [7], [11]). The symmetry is not essential for our arguments, however.

income is bounded away from zero, while, as in the previous sections, the share of capital income tends to zero. This is the case independently of whether there is a sector subject to natural limits to growth and of whether land is required as a factor of production in this sector. The crucial (and realistic) assumption is that land directly enters the utility functions. In the absence of direct services the case with land in the utility function is much like the case with flexible labor supply, mentioned in section 2. The difference is that land incomes are taken into account in most measures of actual income to a larger extent than the opportunity cost of leisure.

In any of these cases landlords, unlike those that only own machines, can in fact remain rich without working.

Empirically it is difficult (although not impossible) to disentangle shares to reproducible and non-reproducible capital. Unlike labor and capital, land and capital are traded on closely related markets. Capitalists remain rich when landlords remain rich if they arbitrage on the same markets and if they have unbiased foresights. If we include the rental incomes from land into those from capital, then our conclusions concerning the evolution of the capital share do no longer hold. However, at this stage it seems questionable whether the rising share of land in capital incomes alone explains can explain the fact that the share of capital does not fall in the course of development. To be able to address the question in a satisfactory way one should reexamine the evolution of factor shares with at least three broad groups of commodities distinguishing between reproducible and non-reproducible capital.

7. APPENDIX: ENDOGENOUS PROPENSITY TO SAVE

We return to the model of Section 3. Rather then assuming that there are workers that do not save and capitalists that save at a constant fraction of their capital incomes, we now assume that there is a representative consumer that chooses the stream of consumption of the two commodities and of savings such as to maximize an intertemporal utility function subject to life time budget.

His problem is to choose the path $C(t) = (Y(t), D(t))$ to maximize

$$\int_0^\infty e^{-\rho t} u(C(t)) dt,$$

where $u(\cdot)$ is the utility function of Section , subject to

$$\begin{aligned} \dot{K}(t) &= w_t(1 - D(t)) + r_t K(t) - Y(t)\dot{K}(t) \\ K(0) &= K_0 > 0 \\ 0 &\leq \lim_{t\to\infty} K(t) e^{\overline{r}(t)t}. \end{aligned}$$

We will show that L_Y, employment in the industrial sector tends to zero. The conclusions about factor shares follow as in the main text.

The current value Hamiltonian of the worker's problem is $\widehat{H}(C,K,t,\mu) = u(C) + q(t)[w_t(1-\dot{D}) + r_tK - Y]$. The FOC's yield $\widehat{H}_Y = 0 \Rightarrow u_Y = q$, $\widehat{H}_D = 0 \Rightarrow u_D = wq$,and $\widehat{H}_K = \rho q - \dot{q} \Rightarrow qr = \rho q - \dot{q} \Rightarrow \rho - r = (\dot{q}/q)$.

Since $u(C) = (Y^\rho + D^\rho)^{\frac{1}{\rho}}$, we get $(\dot{u}_Y/u_Y) = (\dot{q}/q)$, $(\dot{u}_D/u_D) = \frac{\dot{q}w+q\dot{w}}{qw} \Rightarrow \frac{\dot{u}_D}{u_D} = \frac{\dot{q}}{q} + \frac{\dot{w}}{w}$, or $\frac{\dot{u}_D}{u_D} - \frac{\dot{u}_Y}{u_Y} = \frac{\dot{w}}{w}$. We now calculate $\frac{\dot{u}_D}{u_D} - \frac{\dot{u}_Y}{u_Y}$.

Some steps of computation yield $\frac{\dot{u}_Y}{u_Y} = (1-\gamma)[\frac{Y^\gamma}{Y^\gamma+D^\gamma}\frac{\dot{Y}}{Y} + \frac{D^\gamma}{Y^\gamma+D^\gamma}\frac{\dot{D}}{D} - \frac{\dot{Y}}{Y}]$ and $\frac{\dot{u}_D}{u_D} = (1-\gamma)[\frac{Y^\gamma}{Y^\gamma+D^\gamma}\frac{\dot{Y}}{Y} + \frac{D^\gamma}{Y^\gamma+D^\gamma}\frac{\dot{D}}{D} - \frac{\dot{D}}{D}]$. Therefore $\frac{\dot{w}}{w} = \frac{\dot{u}_D}{u_D} - \frac{\dot{u}_Y}{u_Y} = (1-\gamma)[\frac{\dot{Y}}{Y} - \frac{\dot{D}}{D}]$.

Since $D \leq 1$, we must have that $\lim_t \frac{\dot{D}}{D} \leq 0$. Thus we get

$$\lim_t \frac{\dot{w}}{w} \geq (1-\gamma)\lim_t \frac{\dot{Y}}{Y}. \tag{1}$$

On the other hand we have $w = A(f(k) - kf'(k))$ and $Y = AL_Yf(k)$. If k is bounded (see below) then $\lim_t \frac{\dot{w}}{w} \leq \lim_t \hat{A}_t = \hat{A}$. Since $\lim_t \widehat{Y}_t = \hat{A} + \lim_t \widehat{L}_Y$ it follows by 1 that $\hat{A} \geq (1-\gamma)\hat{A} + (1-\gamma)\lim_t \widehat{L}_Y$ and hence $\lim_t \widehat{L}_Y \leq \frac{\gamma}{1-\gamma}\hat{A} < 0$, since $\gamma < 0$.

Thus we have shown that L_Y tends to zero if k_t is bounded.

It remains to show that L_Y tends to zero too if k_t grows without bound. Suppose that $L_Y \to 0$ and $k_t \to \infty$. Then $\widehat{K} = s(\frac{f(k)}{k} + \frac{wD}{K}) - \delta$, where s is the (endogenous) saving rate. Therefore $\lim_t \widehat{K} = \lim_t s\frac{wD}{K} - \delta = \lim_t s\frac{A}{K}(f(k) - kf'(k))D = \lim_t \frac{s}{L_Y}(\frac{f(k)}{k} - f'(k))D = 0$ since $s \leq 1$ and L_Y bounded away from zero. Thus, $\lim_t \widehat{k} < -\lim_t \widehat{AL_Y}$, or $\lim_t \widehat{k} < -\lim_t \widehat{A}$ since L_Y is bounded away from zero. Thus k_t tends to zero, which is a contradiction.

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