

Politically feasible reforms of non-linear tax systems*

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Abstract

We present a conceptual framework for the analysis of politically feasible tax reforms. First, we prove a median voter theorem for monotonic reforms of non-linear tax systems. This yields a characterization of reforms that are preferred by a majority of individuals over the status quo and hence politically feasible. Second, we show that every Pareto-efficient tax system is such that moving towards lower tax rates for below-median incomes and towards higher rates for above median incomes is politically feasible. Third, we develop a method for diagnosing whether a given tax system admits reforms that are welfare-improving and/ or politically feasible.

Keywords: Non-linear income taxation; Tax reforms; Political economy, Welfare analysis.

JEL classification: C72; D72; D82; H21.

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1 Introduction

We study reforms of non-linear income tax systems from a political economy perspective. Starting from a given status quo we characterize reforms that are politically feasible in the sense that a majority of taxpayers will be made better off. In addition, we relate the set of politically feasible reforms to the set of welfare-improving reforms. We thereby introduce a conceptual framework that can be used to analyze whether welfare-improving reforms have a chance in the political process. This framework can also be used to check whether a given tax system is efficient in the sense that the scope for politically feasible welfare improvement has been exhausted, or whether there is room for welfare-improvements that will be supported by a majority of taxpayers.

The analysis of politically feasible reforms is made tractable by focussing on monotonic reforms, i.e. on reforms such that the change in tax payments is a monotonic function of income. We prove a median voter theorem according to which a monotonic reform is politically feasible if and only if it is supported by the taxpayer with median income. The key insight that enables a proof of the theorem is that the Spence-Mirrlees single crossing property translates into a single-crossing property of preferences over monotonic reforms. Thus, if the median voter benefits from a monotonic reform, then all taxpayers with a higher or all tax payers with a lower income will also benefit, implying majority-support for the reform.

Monotonic reforms play a prominent role in the theory of welfare-maximizing taxation. Characterizations of optimal tax systems via the perturbation method often look at the welfare implications of reforms that are monotonic (typically raising the marginal tax rate in a small band of incomes). A welfare-maximizing tax system then has the property that no such reform yields a welfare improvement. By relating our analysis of politically feasible reforms to this approach we can look at the intersection of politically feasible and welfare-improving reforms. We also argue that observed tax reforms are typically monotonic reforms and provide examples from the US, Germany and France.

A simple graphical analysis enables us to diagnose whether a given status quo tax system can be reformed in a politically feasible or welfare-improving way. We derive sufficient statistics that admit a characterization in terms of auxiliary tax schedules. To see whether, say, an increase of marginal tax rates for incomes in a certain range would be politically feasible we can simply look at a graph that plots the status quo and an auxiliary tax schedule for politically feasible reforms. If, for the given range of incomes, tax rates in the status quo are below those stipulated by the auxiliary schedule, then moving towards higher marginal tax rates is politically feasible. If they lie above, moving towards lower rates will be politically feasible. We can use this approach also to diagnose whether there is scope for revenue-increasing, Pareto-improving or welfare-improving reforms.

The characterization of politically feasible reforms shows that there are two Pareto bounds for marginal tax rates. If marginal tax rates in the status quo exceed the upper

bound then lowering tax rates is Pareto-improving and hence politically feasible. Similarly, if marginal tax rates fall short of the lower bound then an increase of tax rates is Pareto-improving and hence politically feasible. If the status quo is Pareto-efficient in the sense that marginal tax rates lie between those bounds, then a reform that raises marginal tax rates for above median incomes and reforms that lower marginal tax rates for below median incomes are politically feasible.

The marked discontinuity at the median level of income provides a possible explanation for the observation that actual tax schedules often have a pronounced increase of marginal tax rates close to the median income: if political economy forces push towards low tax rates below the median and towards high tax rates above the median, then there has to be an intermediate range that connects the low rates below the median with the high rates above the median.

Our derivation of sufficient statistics for politically feasible or welfare-improving reforms is based on an analysis of reforms that involve a change of marginal tax rates that applies only to incomes in a certain bracket. More specifically, it is based on an analysis of small reforms. They are small in that we look at the implications of a marginal change of tax rates applied to a bracket of incomes with vanishing length. However, our analysis is explicit about the transition from a large reform that involves a discrete change of marginal tax rates applied to a non-negligible range of incomes to a reform that involves only a marginal change of tax rates, but applied to a non-negligible range of incomes, and, finally, to a reform that involves a marginal change of tax rates for a negligible range of incomes. We believe that this derivation is of pedagogical value. It complements both the heuristic approaches due to Piketty (1997) and Saez (2001) and approaches that make use of functional derivatives such as Golosov, Tsyvinski and Werquin (2014) and Jacquet and Lehmann (2016).

The remainder is organized as follows. The next section discusses related literature. The formal framework is introduced in Section 3. Our analysis is based on a generic Mirrleesian model of income taxation. Section 4 presents median voter theorems for monotonic reforms. The characterization of politically feasible and welfare improving reforms by means of sufficient statistics can be found in Section 5. Section 6 shows that the median voter theorem for monotonic reforms extends to models of taxation that are richer than the basic setup due to Mirrlees (1971). Specifically, we consider the possibility to mix direct and indirect taxes as in Atkinson and Stiglitz (1976), the possibility to add sources of heterogeneity among individuals such as fixed costs of labor market participation or public goods preferences, and the possibility that taxpayers seek to mitigate income differences that are due to luck as opposed to effort, as in Alesina and Angeletos (2005). The last section contains concluding remarks. Unless stated otherwise, proofs are relegated to the Appendix.

2 Related literature

The Mirrleesian framework is the workhorse for the normative analysis of non-linear tax systems, see Hellwig (2007) and Scheuer and Werning (2016) for more recent analyses of this model and Piketty and Saez (2013) for a literature review.

Saez and Stantcheva (2016) study generalized welfare functions with weights that need not be consistent with the maximization of a utilitarian social welfare function. The generalized weights may as well reflect alternative, non-utilitarian value judgments or political economy forces. Saez and Stantcheva emphasize the similarities between utilitarian welfare maximization and political economy considerations: both can be represented as resulting from the maximization of a generalized welfare function. Our approach takes an alternative route and emphasizes the differences between the requirements of political feasibility and welfare maximization. We distinguish the set of politically feasible reforms from the set of welfare-improving reforms so as to be able to provide possibility and impossibility results for politically feasible welfare-improvements.

Well-known political economy approaches to redistributive income taxation have used the model of linear income taxation due to Sheshinski (1972). In this model, marginal tax rates are the same for all levels of income and the resulting tax revenue is paid out as uniform lump-sum transfers. As has been shown by Roberts (1977), the median voter's preferred system is a Condorcet winner in the set of all linear income tax systems (see, e.g., Drazen, 2000; Persson and Tabellini, 2000). Our finding that a median voter theorem applies to monotonic reforms of non-linear tax systems generalizes results by Rothstein (1990; 1991) and Gans and Smart (1996) who focussed on linear tax systems.

Median voter theorems for linear income taxation have been widely used on the assumption that voters are selfish. A prominent example is the prediction due to Meltzer and Richard (1981) that tax rates are an increasing function of the difference between median and average income. The explanatory power of this framework was found to be limited – see, for instance, the review in Acemoglu, Naidu, Restrepo and Robinson (2015) – and has led to analyses in which the preferences for redistributive tax policies are also shaped by prospects for upward mobility or a desire for a fair distribution of incomes.¹ In Section 6 we extend our basic analysis and prove a median voter theorem for reforms of non-linear tax systems that takes account of such demands for fairness.

We derive sufficient statistics that enable us to identify reforms that are in the median voter's interest and would therefore be supported by a majority of voters. The relevant sufficient statistics takes the form of an auxiliary tax schedule. The auxiliary schedule in turn has properties which are similar to the work of Röell (2012) and Brett and Weymark (2017; 2016) who characterize the non-linear income tax schedule that the median voter would pick if she could dictate tax policy. Both schedules reveal that the median voter

¹See, for instance, Piketty (1995), Bénabou and Ok (2001), Alesina and Angeletos (2005), Bénabou and Tirole (2006), or Alesina, Stantcheva and Teso (2017).

wants to have low taxes on the poor and high taxes on the rich. For the special case of quasi-linear utility functions, the auxiliary schedule indeed coincides with the median voter’s preferred schedule whenever the latter does not give rise to bunching.

A classical idea in public finance is that the theory of optimal taxation – which characterizes welfare-maximizing tax systems and has no role for current tax policy – should be complemented by a theory of tax reforms with a focus on incremental changes that apply to a given status quo, see Feldstein (1976). Weymark (1981) studies the scope for Pareto-improving reforms of a commodity tax system. To give another example, Guesnerie (1995) contains an analysis of tax reforms that emphasizes political economy forces, formalized as a requirement of coalition-proofness, that affect the implementability of reforms. Our analysis contributes to this earlier literature by combining results from social choice theory on the applicability of median voter theorems with the perturbation approach to the analysis of non-linear tax systems. Our main results in Theorems 1 and 2 provide a characterization of politically feasible reforms of non-linear tax systems. Getting there requires arguments from both strands of the literature.

The focus on the conditions under which a status quo tax policy admits reforms that are politically feasible distinguishes our work from papers that explicitly analyze political competition as a strategic game and then characterize equilibrium tax policies.²

Our analysis of welfare-improving reforms can be related to a literature that seeks to identify society’s social welfare function empirically.³ Through the lens of our model, this literature can alternatively be interpreted as identifying the set of social welfare functions for which a given reform – e.g. an increase of marginal tax rates for incomes close to the sixtieth-percentile of the income distribution – would be welfare-improving.

3 The model

3.1 Preferences

There is a continuum of individuals of measure 1. Individuals are confronted with a predetermined income tax schedule T_0 that assigns a (possibly negative) tax payment

²Examples include Acemoglu, Golosov and Tsyvinski (2008; 2010) who relate dynamic problems of optimal taxation to problems of political agency as in Barro (1973) and Ferejohn (1986); Farhi, Sleet, Werning and Yeltekin (2012) and Scheuer and Wolitzky (2016) who study optimal capital taxation subject to the constraints from probabilistic voting as in Lindbeck and Weibull (1987); Battaglini and Coate (2008) who study optimal taxation and debt financing in a federal system using the model of legislative bargaining due to Baron and Ferejohn (1989); Bierbrauer and Boyer (2016) who study Downsian competition with a policy space that includes non-linear tax schedules and possibilities for pork-barrel spending as in Myerson (1993). Ilzetzki (2015) studies reforms of the commodity tax system using a model of special interests politics.

³See, for instance, Christiansen and Jansen (1978), Blundell, Brewer, Haan and Shephard (2009), Bourguignon and Spadaro (2012), Bargain, Dolls, Neumann, Peichl and Siegloch (2011), Hendren (2014), Zoutman, Jacobs and Jongen (2014), Lockwood and Weinzierl (2016), or Bastani and Lundberg (2016).

$T_0(y)$ to every level of pre-tax income $y \in \mathbb{R}_+$. Under the initial tax system individuals with no income receive a transfer equal to $c_0 \geq 0$. We assume that T_0 is everywhere differentiable so that marginal tax rates are well-defined for all levels of income. We also assume that $y - T_0(y)$ is a non-decreasing function of y and that $T_0(0) = 0$.

Individuals have a utility function u that is increasing in private goods consumption, or after-tax income, c , and decreasing in earnings or pre-tax income y . Utility also depends on a measure of the individual's productive ability, referred to as the individual's type. The set of possible types is denoted by Ω and taken to be a compact subset of the non-negative real numbers, $\Omega = [\underline{\omega}, \bar{\omega}] \subset \mathbb{R}_+$. A typical element of Ω will be denoted by ω . The utility that an individual with type ω derives from c and y is denoted by $u(c, y, \omega)$. The cross-section distribution of types in the population is represented by a cumulative distribution function F with density f . We write ω^x for the skill type with $F(\omega^x) = x$. For $x = \frac{1}{2}$, this yields the median skill type also denoted by ω^M .

The slope of an individual's indifference curve in a y - c -diagram $-\frac{u_y(c, y, \omega)}{u_c(c, y, \omega)}$ measures how much extra consumption an individual requires as a compensation for a marginally increased level of pre-tax income. We assume that this quantity is decreasing in the individual's type, i.e. for any pair (c, y) and any pair (ω, ω') with $\omega' > \omega$,

$$-\frac{u_y(c, y, \omega')}{u_c(c, y, \omega')} \leq -\frac{u_y(c, y, \omega)}{u_c(c, y, \omega)}.$$

This assumption is commonly referred to as the *Spence-Mirrlees single crossing property*, see Figure 5 (left panel).

Occasionally, we illustrate our results by looking at more specific utility functions. One case of interest is the specification of a utility function $U : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ so that $u(c, y, \omega) = U(c, \frac{y}{\omega})$. We can then interpret ω as an hourly wage and $l = \frac{y}{\omega}$ as the time that an individual needs to generate a pre-tax-income of y . Another case of interest is the quasi-linear in private goods consumption specification so that $u(c, y, \omega) = c - k(y, \omega)$. The function k then gives the cost of productive effort. If we combine both cases utility can be written as $c - \tilde{k}(\frac{y}{\omega})$, where the function \tilde{k} is taken to be increasing and convex. The cost function \tilde{k} is said to be iso-elastic if it takes the form $\tilde{k}(\frac{y}{\omega}) = (\frac{y}{\omega})^{1+\frac{1}{\epsilon}}$, for some parameter $\epsilon > 0$.

We assume that leisure is a non-inferior good. If individuals experience an increase in an exogenous source of income e , they do not become more eager to work. More formally, we assume that for any pair (c, y) any ω and any $e' > e$,

$$-\frac{u_y(c+e, y, \omega)}{u_c(c+e, y, \omega)} \leq -\frac{u_y(c+e', y, \omega)}{u_c(c+e', y, \omega)}.$$

We can also express this condition by requiring that, for any combination of c, y, e and ω , the derivative of $-\frac{u_y(c+e, y, \omega)}{u_c(c+e, y, \omega)}$ with respect to e is non-negative. This yields the following condition: for all c, y, e and ω ,

$$-u_{cc}(c+e, y, \omega) \frac{u_y(c+e, y, \omega)}{u_c(c+e, y, \omega)} + u_{cy}(c+e, y, \omega) \leq 0. \quad (1)$$

Finally, we assume that an individual's marginal utility of consumption $u_c(c, y, \omega)$ is both non-increasing in c and non-increasing in ω , i.e. $u_{cc}(c, y, \omega) \leq 0$ and $u_{c\omega}(c, y, \omega) \leq 0$. These assumptions hold for any utility function $u(c, y, \omega) = v(c) - k(y, \omega)$ that is additively separable between private goods consumption c on the one hand and the pair (y, ω) on the other, where v is a (weakly) concave function. With $u(c, y, \omega) = U\left(c, \frac{y}{\omega}\right)$, $u_{c\omega}(c, y, \omega) \leq 0$ holds provided that $U_{cl}\left(c, \frac{y}{\omega}\right) \geq 0$ so that working harder makes you more eager to consume.⁴

3.2 Reforms

A reform induces a new tax schedule T_1 that is derived from T_0 so that, for any level of pre-tax income y , $T_1(y) = T_0(y) + \tau h(y)$, where τ is a scalar and h is a function. We represent a reform by the pair (τ, h) . Without loss of generality, we focus on reforms such that $y - T_1(y)$ is non-decreasing. The reform induces a change in tax revenue denoted by $\Delta^R(\tau, h)$. For now we assume that this additional tax revenue is used to increase the basic consumption level c_0 . Alternatives are considered in Section 6.

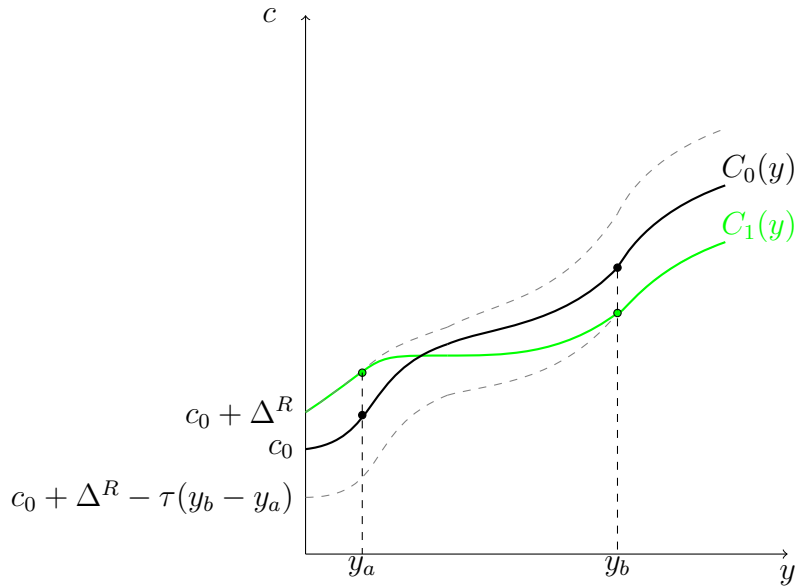


Figure 1: A reform in the (τ, y_a, y_b) -class

Some of our results follow from looking at a special class of reforms. For this class, there exists a first threshold level of income y_a , so that the new and the old tax schedule coincide for all income levels below the threshold, $T_0(y) = T_1(y)$ for all $y \leq y_a$. There exists a second threshold $y_b > y_a$ so that for all incomes between y_a and y_b marginal tax rates are increased by τ , $T_0'(y) + \tau = T_1'(y)$ for all $y \in (y_a, y_b)$. For all incomes above y_b , marginal tax rates coincide, so that $T_0'(y) = T_1'(y)$ for all $y \geq y_b$. Hence, the function h

⁴Seade (1982) refers to $U_{cl}\left(c, \frac{y}{\omega}\right) \geq 0$ as non-Edgeworth complementarity of leisure and consumption.

is such that

$$h(y) = \begin{cases} 0, & \text{if } y \leq y_a, \\ y - y_a, & \text{if } y_a < y < y_b, \\ y_b - y_a, & \text{if } y \geq y_b. \end{cases}$$

For reforms of this type we will write (τ, y_a, y_b) rather than (τ, h) . Figure 1 shows how a reform in the (τ, y_a, y_b) -class affects the combinations of consumption c and earnings y that are available to individuals. Specifically, the figure shows the curves

$$C_0(y) = c_0 + y - T_0(y), \quad \text{and} \quad C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau h(y).$$

Part A of the Appendix contains a detailed analysis of the behavioral responses to such reforms that takes account of the bunching that results from the kink in the tax schedule generated by the reform.

Reforms of the (τ, y_a, y_b) -type play a prominent role in the literature. Papers that use functional derivatives to analyze tax perturbations rest, however, on the assumption that both pre- and post-reform earnings are characterized by first order conditions. Reforms in the (τ, y_a, y_b) -class can not be approached directly with this approach. Instead one looks at smooth reforms that approximate such reforms. The formal analysis that follows applies both to reforms in the (τ, y_a, y_b) -class and to reforms where pre- and post-reform earnings follow from first order conditions. In the remainder and without further mention we focus on these two classes of reforms.

Notation. To describe the implications of reforms for measures of revenue, welfare and political support it proves useful to introduce the following optimization problem: choose y so as to maximize

$$u(c_0 + e + y - T_0(y) - \tau h(y), y, \omega), \quad (2)$$

where e is a source of income that is exogenous from the individual's perspective. We assume that this optimization problem has, for each type ω , a unique solution that we denote by $y^*(e, \tau, \omega)$.⁵ The corresponding indirect utility level is denoted by $V(e, \tau, \omega)$. Armed with this notation we can express the reform-induced change in tax revenue as

$$\Delta^R(\tau, h) := \int_{\underline{\omega}}^{\bar{\omega}} \{T_1(y^*(\Delta^R(\tau, h), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega.$$

The reform-induced change in indirect utility for a type ω individual is given by

$$\Delta^V(\omega | \tau, h) := V(\Delta^R(\tau, h), \tau, \omega) - V(0, 0, \omega).$$

⁵For ease of exposition, we ignore the non-negativity constraint on y in the body of the text and relegate this extension to part C in the Appendix. There, we clarify how the analysis has to be modified if there is a set of unemployed individuals whose labor market participation might be affected by a reform.

Pareto-improving reforms. A reform (τ, h) is said to be Pareto-improving if, for all $\omega \in \Omega$, $\Delta^V(\omega | \tau, h) \geq 0$, and if this inequality is strict for some $\omega \in \Omega$.

Welfare-improving reforms. We consider a class of social welfare functions. Members of this class differ with respect to the specification of welfare weights. Admissible welfare weights are represented by a non-increasing function $g : \Omega \rightarrow \mathbb{R}_+$ with the property that the average welfare weight equals 1, $\int_{\underline{\omega}}^{\bar{\omega}} g(\omega) f(\omega) d\omega = 1$. We denote by $G(\omega) := \int_{\omega}^{\bar{\omega}} g(s) \frac{f(s)}{1-F(\omega)} ds$ the average welfare weight among individuals with types above ω . Note that, if g is strictly decreasing, then $G(\omega) < 1$, for all $\omega > \underline{\omega}$. For a given function g , the welfare change that is induced by a reform is given by

$$\Delta^W(g | \tau, h) := \int_{\underline{\omega}}^{\bar{\omega}} g(\omega) \Delta^V(\omega | \tau, h) f(\omega) d\omega .$$

A reform (τ, h) is said to be welfare-improving if $\Delta^W(g | \tau, h) > 0$.

Political support for reforms. Political support for the reform is measured by the mass of individuals who are made better if the initial tax schedule T_0 is replaced by T_1 ,

$$S(\tau, h) := \int_{\underline{\omega}}^{\bar{\omega}} \mathbf{1}\{\Delta^V(\omega | \tau, h) > 0\} f(\omega) d\omega ,$$

where $\mathbf{1}\{\cdot\}$ is the indicator function. A reform (τ, h) is said to be supported by a majority of the population if $S(\tau, h) \geq \frac{1}{2}$.

3.3 Types and earnings

Earnings depend inter alia on the individuals' types in our framework. However, observed tax policies express marginal tax rates as a function of income. To relate the results from our theoretical analysis to observed tax policies we repeatedly invoke the function $\tilde{y}^0 : \Omega \rightarrow \mathbb{R}_+$ where $\tilde{y}^0(\omega) = y^*(0, 0, \omega)$ gives earnings as a function of type in the status quo. We denote the inverse of this function by $\tilde{\omega}^0$ so that $\tilde{\omega}^0(y)$ is the type who earns an income of y in the status quo.

By the Spence-Mirrlees single crossing property the function \tilde{y}^0 is weakly increasing. The existence of its inverse $\tilde{\omega}^0$ requires in addition that, under the status quo schedule T_0 , there is no bunching so that different types choose different levels of earnings.

Assumption 1 *The function \tilde{y}^0 is strictly increasing.*

It is not difficult to relax this assumption. However, taking account of bunching in the status quo requires additional steps in the formal analysis that we relegate to part C of the Appendix. This extension is relevant because empirically observed tax schedules frequently have kinks and hence give rise to bunching, see e.g. Saez (2010) and Kleven (2016).

In principle, the functions \tilde{y}^0 and $\tilde{\omega}^0$ could be estimated from any data set that contains both data on individual earnings and productive abilities. In the original analysis of Mirrlees (1971) hourly wages are taken to be the measure of productive abilities. Our framework is consistent with this approach, but is also compatible with other measures of ability.

When we illustrate our formal results by means of examples, we will assume that both the distributions of earnings and the distribution of productive abilities are Pareto distributions.⁶ Under these assumptions, earnings are an iso-elastic function of types, see the following remark.

Remark 1 *Suppose that the type distribution is a Pareto distribution that is represented by the cdf $F(\omega) = 1 - \left(\frac{\omega_{\min}}{\omega}\right)^a$. Also suppose that, under the status quo tax schedule T_0 , the earnings distribution is a Pareto-distribution with cdf $\tilde{F}^0(y) = 1 - \left(\frac{y_{\min}}{y}\right)^b$, where $y_{\min} > 0$ and $\omega_{\min} > 0$. Then, for some multiplicative constant α ,*

$$\tilde{y}^0(\omega) = \alpha \omega^\gamma \quad \text{and} \quad \tilde{\omega}^0(y) = \left(\frac{y}{\alpha}\right)^{\frac{1}{\gamma}}, \quad \text{where} \quad \gamma = \frac{a}{b}.$$

4 Median voter theorems for monotonic reforms

4.1 Monotonic reforms

A tax reform (τ, h) is said to be monotonic over a range of incomes $\mathcal{Y} \subset \mathbb{R}_+$ if

$$T_1(y) - T_0(y) = \tau h(y)$$

is a monotonic function for $y \in \mathcal{Y}$. Obviously, this is the case if h is monotonic for $y \in \mathcal{Y}$. We say that a reform is monotonic if h is monotonic over \mathbb{R}_+ . Given a cross-section distribution of income, we say that a reform is monotonic above (below) the median if $h(y)$ is a monotonic function for incomes above (below) the median income.

Monotonic reforms in research on tax systems. Monotonic reforms play a prominent role in the literature. For instance, the characterization of a welfare-maximizing tax systems in Saez (2001) is based on reforms in the (τ, y_a, y_b) -class. This is a class of monotonic reforms. Political economy analysis in the tradition of Roberts (1977) and Meltzer and Richard (1981) focus on linear tax systems. A reform of a linear income tax system can be described as a pair (τ, h) with $h(y) = y$ so that $T_0(y) - T_1(y) = \tau y$. Again, h is a monotonic function. Heathcote, Storesletten and Violante (forthcoming) focus on income tax systems in a class that has a constant rate of progressivity. A tax system

⁶Diamond (1998) takes hourly wages as a measure of productive abilities and shows that relevant parts of the wage distribution are well approximated by a Pareto-distribution. Saez (2001) and Diamond and Saez (2011) argue that relevant parts of the earnings distribution are also well approximated by a Pareto distribution.

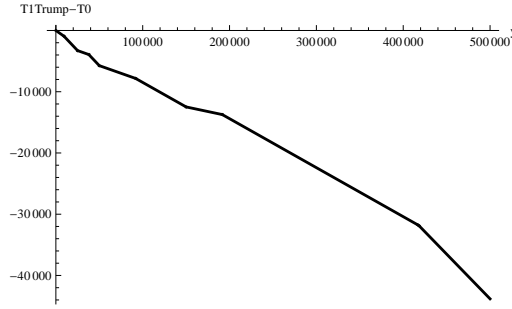


Figure 2: Donald Trump’s reform proposal for the federal income tax

Figure 2 shows, for each level of earnings, the tax cut that would result if the pre-election US federal income tax T_0 were replaced by Donald Trump’s proposal T_1^{TRUMP} . After the reform, taxes would be lower for all earnings levels. Moreover, the tax cut would be increasing in earnings. The figure is based on the information provided in *Republican Blueprint*, <https://abetterway.speaker.gov/assets/pdf/ABetterWay-Tax-PolicyPaper.pdf>, retrieved on 18 April 2017.

in this class takes the form $T(y) = y - \lambda y^{1-\rho}$ where the parameter ρ is the measure of progressivity and the parameter λ affects the level of taxation. An increase of the rate of progressivity can be viewed as a reform (τ, h) so that h is increasing for incomes above a threshold \hat{y} and decreasing for incomes below \hat{y} .⁷ As a consequence, such a reform is monotonic either below or above the median.

Monotonic reforms and tax policy. Monotonic reform proposals are also prominent in the political process. For instance, in the campaign for the 2016 US presidential election Donald Trump’s tax plan included a change in the federal income tax so that there would be tax cuts for all levels of income, i.e. $T_1(y) - T_0(y) < 0$, for all y . Moreover, under Trump’s proposal, $T_1(y) - T_0(y)$ is monotonically decreasing so that tax cuts are larger for larger levels of income, see Figure 2.

The reforms of the German income tax system that were proposed prior to the federal election in 2013 were all monotonic below the median (see Figure 3): the far left and the green party proposed changes of the tax code that involved cuts for incomes below a threshold and higher taxes for incomes above it. The threshold was above the median and $T_1(y) - T_0(y)$ was decreasing for below median incomes. The proposal of the social democrats only involved higher tax for the very rich. Under their proposal $T_1(y) - T_0(y)$ was both non-negative and non-decreasing, for all y . Like the Trump plan, the liberal party proposed tax cuts that that were increasing in income. The conservative party did not propose any change of the income tax schedule.

We investigated the whole sequence of annual changes in the French income tax sched-

⁷Formally, let the status quo be a tax system T_0 with $T_0(y) = y - \lambda_0 y^{1-\rho_0}$. Consider the move to a new tax system T_1 with $T_1(y) = y - \lambda_1 y^{1-\rho_1}$ and $\rho_1 > \rho_0$. Then $\tau h(y) = T_1(y) - T_0(y) = \lambda_0 y^{1-\rho_0} - \lambda_1 y^{1-\rho_1}$. This expression is strictly increasing in y for $y > \hat{y} := \left(\frac{\lambda_1(1-\rho_1)}{\lambda_0(1-\rho_0)} \right)^{\frac{1}{\rho_1-\rho_0}}$ and strictly decreasing for $y < \hat{y}$.

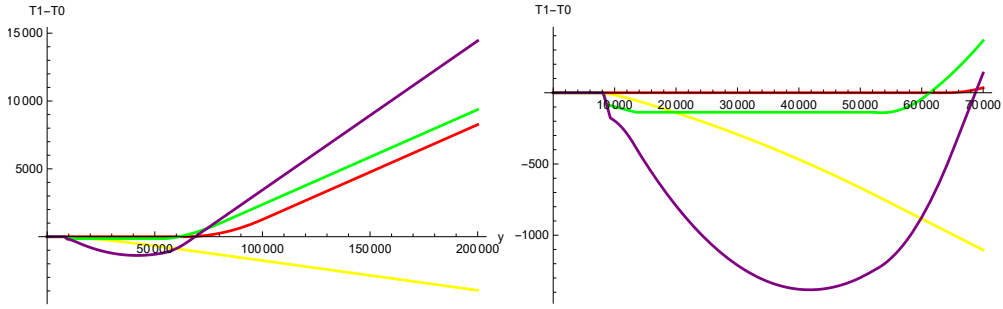


Figure 3: Reforms of the federal income tax proposed prior to Germany’s 2013 election. Figure 3 (left panel) shows, for each level of earnings, the tax cut or raise that would have resulted if the pre-election income tax T_0 had been replaced by the proposal of the social democratic party (SPD, red), the far left (Die Linke, purple), the green party (Bündnis 90-Die Grünen, green), or the liberal party (FDP, yellow). The conservative party (CDU) did not propose changes of the income tax and is hence associated with the horizontal axis. Figure 3 (right panel) zooms on annual earnings below 70,000 Euros. The figure is based on the information provided in Peichl, Pestel, Sieglöckh and Sommer (2014).

ule since World War II. There are adjustments almost every year. In most cases the changes in tax payments were monotonic for all incomes.⁸ Figure 4 (left panel) displays major reforms from recent years that were monotonic for all incomes and one example (right panel) of a reform that was monotonic below the median.

4.2 Small reforms

We begin with an analysis of small reforms and turn to large reforms subsequently. We say that an individual of type ω benefits from a small reform if, at $\tau = 0$,

$$\Delta_{\tau}^V(\omega \mid \tau, h) := \frac{d}{d\tau} V(\Delta^R(\tau, h), \tau, \omega) > 0 .$$

Theorem 1 *Let h be a monotonic function. The following statements are equivalent:*

1. *The median voter benefits from a small reform.*
2. *There is a majority of voters who benefit from a small reform.*

To obtain an intuitive understanding of Theorem 1, consider a policy-space in which individuals trade-off increased transfers and increased taxes. The following Lemma provides a characterization of preferences over such reforms.

Lemma 1 *Consider a τ - Δ^R diagram and let $s^0(\omega)$ be the slope of a type ω individual’s indifference curve through point (τ, Δ^R) . For any ω , $s^0(\omega) = h(\tilde{y}^0(\omega))$.*

⁸There are seven exceptions. Under three of those, the changes in tax payments were monotonic below or above the median.

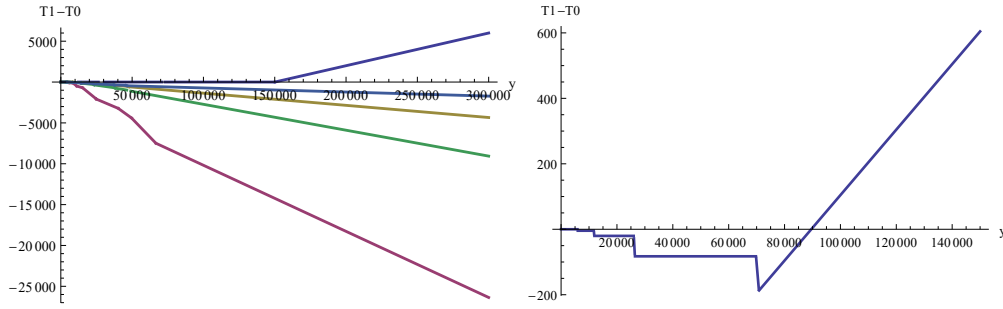


Figure 4: Recent reforms of the French income tax

Figure 4 (left panel) shows some major reforms (in terms of increase or decrease of tax payments). The reforms were implemented in years 2013 (dark blue), 2007 (purple), 2004 (brown), 2003 (green), and 2002 (blue). Figure 4 (right panel) shows the reform implemented in year 2011. The figure is based on the information provided by barèmes IPP, <http://www.ipp.eu/outils/baremes-ipp/>.

Consider, for the purpose of illustration, a reform in the (τ, y_a, y_b) -class. Individuals who choose earnings below y_a are not affected by the increase of tax rates. As a consequence, $s^0(\omega) = h(\tilde{y}^0(\omega)) = 0$ which means that they are indifferent between a tax increase $\tau > 0$ and increased transfers $\Delta^R \geq 0$ only if $\Delta^R = 0$. As soon as $\tau > 0$ and $\Delta^R > 0$, they are no longer indifferent, but benefit from the reform. Individuals with higher levels of income are affected by the increase of the marginal tax rate, and would be made worse off by any reform with $\tau > 0$ and $\Delta^R = 0$. Keeping them indifferent requires $\Delta^R > 0$ as reflected by the observation that $s^0(\omega) = h(\tilde{y}^0(\omega)) > 0$. Moreover, if h is a non-decreasing function of y , the higher an individual's income the larger is the increase in Δ^R that is needed in order to compensate the individual for an increase of marginal tax rates. Noting that $\tilde{y}^0(\omega)$ is a non-decreasing function of ω by the Spence-Mirrlees single crossing property, we obtain the following Corollary to Lemma 1.

Corollary 1 *Suppose that h is a non-decreasing function of y . Then $\omega' > \omega$, implies*

$$\tilde{y}^0(\omega) \leq \tilde{y}^0(\omega') \quad \text{and} \quad s^0(\omega) \leq s^0(\omega').$$

Corollary 1 establishes a single-crossing property for indifference curves in a τ - Δ^R -space, see Figure 5 (right panel). The indifference curve of a richer individual is steeper than the indifference curve of a poorer individual. Thus, if h is a non-decreasing function of y , it is more difficult to convince richer individuals that a reform that involves higher taxes and higher transfers is worthwhile. This is the driving force for the median voter theorem (Theorem 1): if the median voter likes such a reform, then anybody who earns less will also like it so that the supporters of the reform constitute a majority. If the median voter prefers the status quo over the reform, then anybody who earns more also prefers the status quo. Then, the opponents of the reform constitute a majority.

Roberts (1977) also used a single-crossing property to show that the median voter's preferred tax policy is a Condorcet winner in the set of affine tax policies, see the dis-

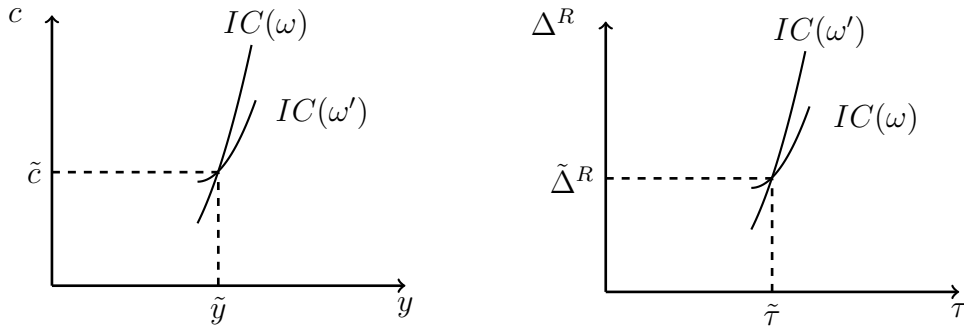


Figure 5: Single-crossing properties

The figure shows indifference curves of two types ω' and ω with $\omega' > \omega$: The panel on the left illustrates the Spence-Mirrlees single-crossing property. In a $y - c$ space lower types have steeper indifference curves as they are less willing to increase their earnings in exchange for a given increase of their consumption level. The panel on the right shows indifference curves in a $\tau - \Delta^R$ space. Here, lower types have flatter indifference curves indicating that they are more willing to accept an increase of marginal taxes in exchange for increased transfers.

cussion in Gans and Smart (1996). Our analysis generalizes these findings in two ways. First, we show that a single crossing-property holds if we consider reforms that move away from some predetermined non-linear tax schedule, and not only to reforms that modify a predetermined affine tax schedule. Second, we do not require that marginal tax rates are increased for all levels of income. For instance, the single-crossing property also holds for reforms where the increase applies only to a subset $[y_a, y_b]$ of all possible income levels.

Theorem 1 exploits only that taxpayers can be ordered according to the slope of their indifference curves in the status quo and that this ranking coincides with a ranking according to income. As is well known, the slopes of indifference curves are unaffected by monotonic transformations of utility functions. Thus, the theorem does not rely on a cardinal interpretation of the utility function u .

The next Corollary also follows from the observation that individual preferences over reforms are monotonic in incomes and therefore monotonic in types by the Spence-Mirrlees single crossing property.

Corollary 2 *Let h be a non-decreasing function.*

1. *A small reform (τ, h) with $\tau > 0$ is Pareto-improving if and only if the most-productive or richest individual is not made worse off, i.e. if and only if $\Delta_\tau^V(\bar{\omega} \mid 0, h) \geq 0$.*
2. *A small reform (τ, h) with $\tau < 0$ is Pareto-improving if and only if the least-productive or poorest individual is not made worse off, i.e. if and only if $\Delta_\tau^V(\underline{\omega} \mid 0, h) \geq 0$.*

3. A small reform (τ, h) with $\tau > 0$ benefits voters in the bottom x per cent and harms voters in the top $1 - x$ per cent if and only if $\Delta_\tau^V(\omega^x | 0, h) = 0$.

Corollary 2 is particularly useful for answering the question whether a given status quo tax policy admits Pareto-improving reforms.

Not all conceivable reforms are such that h is non-decreasing. For instance, a reform that pushes a schedule with marginal tax rates that increase in income towards a flat tax (i.e. towards a schedule with a constant marginal tax rate) will involve higher marginal tax rates for the poor and lower marginal tax rates for the rich. Under such a reform, h is increasing for low incomes and decreasing for high incomes. The following Proposition shows that such a reform may still be politically feasible.

Proposition 1 *Let h be non-increasing for $y \geq y^M$. If the median voter benefits from a small reform, then it is politically feasible.*

If the reform is designed in such a way that the voter with median income benefits from a move towards a flatter tax schedule, then this move is supported by a majority of taxpayers. Proposition 1 covers, in particular, the reform of the federal income tax that has been proposed by Trump (see Figure 2). Trump proposes a reform that is monotonic for all levels of income and hence also for incomes above the median.

We can also consider the case of tax cuts for low incomes and, moreover, suppose that h is non-increasing for incomes below a threshold \hat{y} , as for the proposals of the far left and the green party in Germany's 2013 election. In this case, political feasibility is ensured by putting the threshold above the median, so that everybody with below median income is a beneficiary of the reform. This case is covered by the following Proposition.

Proposition 2 *Let h be non-increasing for $y \leq y^M$. If the poorest voter benefits from a small reform, then it is politically feasible.*

4.3 Large reforms

We can evaluate the gains or losses from large reforms simply by integrating over the gains and losses from small reforms since

$$\Delta^V(\omega | \tau, h) = \int_0^\tau \Delta_s^V(\omega | s, h) ds . \quad (3)$$

We first provide a more general characterization of the function Δ_τ^V . We still evaluate small reforms, but no longer impose that the reform is a departure from the status quo schedule with $\tau = 0$. Instead, we allow for the possibility that τ has already been raised from 0 to some value $\tau' > 0$ and consider the implications of a further increase of τ .

Lemma 2 *For all ω ,*

$$\Delta_\tau^V(\omega | \tau', h) = \tilde{u}_c^1(\omega) (\Delta_{\tau'}^R(\tau', h) - h(\tilde{y}^1(\omega))) , \quad (4)$$

where $\tilde{u}_c^1(\omega) := u_c(c_0 + \Delta^R(\tau', h) + \tilde{y}^1(\omega) - T_1(\tilde{y}^1(\omega)), \tilde{y}^1(\omega), \omega)$ is a shorthand for the marginal utility of consumption that a type ω individual realizes after the reform and $\tilde{y}^1(\omega) := y^*(\Delta^R(\tau', h), \tau', \omega)$ is the corresponding earnings level.

If h is a monotonic function, then a reform's impact on available consumption as measured by $\Delta_\tau^R(\tau', h) - h(\tilde{y}^1(\omega))$ is a monotonic functions of ω . These observations enable us to provide an extension of Theorem 1 to large reforms: if every marginal increase of τ yields a gain $\Delta_\tau^R(\tau', h) - h(\tilde{y}^1(\omega))$ that is larger for less productive types, then a discrete change of τ also yields a gain that is larger for less productive types. As a consequence, if the median voter benefits if τ is raised from zero to some level $\tau' > 0$, then anyone with below-median income will also benefit. If the median voter does not benefit, then anyone with above-median income will also oppose the reform. Consequently, a reform is politically feasible if and only if it is in the median voter's interest.

Proposition 3 *Let h be a monotonic function.*

1. *Consider a reform (τ, h) so that for all $\tau' \in (0, \tau)$, $\Delta_\tau^V(\omega^M | \tau', h) > 0$, then this reform is politically feasible.*
2. *Consider a reform (τ, h) so that for all $\tau' \in (0, \tau)$, $\Delta_\tau^V(\omega^M | \tau', h) < 0$, then this reform is politically infeasible.*

Propositions 1 and 2 also extend to large reforms with similar qualifications.

If we use the utility function u for an interpersonal comparison of utilities – as is standard in the literature on optimal taxation in the tradition of Mirrlees (1971) – we can, for instance, compare the utility gains that “the poor” realize if the tax system is reformed to those that are realized by “middle-class” voters.

Lemma 3 *For any pair (ω, ω') with $\omega < \omega'$, $\tilde{u}_c^1(\omega) \geq \tilde{u}_c^1(\omega')$.*

According to this lemma, individuals with low types – who have less income than high types because of the Spence-Mirrlees single crossing property – are also more deserving in the sense that they benefit more from additional consumption. Utilitarian welfare would therefore increase if “the rich” consumed less and “the poor” consumed more. Together with Lemma 2, Lemma 3 also enables an interpersonal comparison of gains and losses from large tax reforms provided that h is non-decreasing: consider a type ω' and a reform (τ, h) so that for all $\tau' \in (0, \tau)$, $\Delta_\tau^V(\omega' | \tau', h) > 0$. The utility gain of a type ω' individual is given by

$$\begin{aligned} \Delta^V(\omega' | \tau, h) &= \int_0^\tau \Delta_\tau^V(\omega' | s, h) ds \\ &= \int_0^\tau \tilde{u}_c^1(\omega') (\Delta_\tau^R(s, h) - h(\tilde{y}^1(\omega'))) ds. \end{aligned}$$

Now consider an individual with type $\omega < \omega'$. Then $\tilde{u}_c^1(\omega) \geq \tilde{u}_c^1(\omega')$ and $\tilde{y}^1(\omega) \leq \tilde{y}^1(\omega')$ imply

$$\int_0^\tau \tilde{u}_c^1(\omega) (\Delta_\tau^R(s, h) - h(\tilde{y}^1(\omega))) ds \geq \int_0^\tau \tilde{u}_c^1(\omega') (\Delta_\tau^R(s, h) - h(\tilde{y}^1(\omega'))) ds$$

and hence $\Delta^V(\omega \mid \tau, h) \geq \Delta^V(\omega' \mid \tau, h)$, i.e. low types realize larger utility gains than high types.

5 Detecting politically feasible reforms

We focus on small reforms in the (τ, y_a, y_b) -class and provide a characterization of politically feasible reforms. We will also discuss the relation between politically feasible reforms and welfare-improving reforms. Political feasibility requires that a reform makes a sufficiently large number of individuals better off. Welfare considerations, by contrast, trade-off utility gains and losses of different individuals. A reform that yields high gains to a small group of individuals and comes with small losses for a large group can be welfare-improving, but will not be politically feasible. A reform that has small gains for many and large costs for few might be politically feasible, but will not be welfare-improving. Our analysis in this section will provide a characterization of tax schedules that can be reformed in such a way that the requirements of political feasibility and welfare improvements are both met. We also provide a characterization of tax schedules that are constrained efficient in the sense that they leave no room for welfare-improvements that are politically feasible.

5.1 Pareto-efficient tax systems and politically feasible reforms

We provide a characterization of Pareto-efficient tax systems that clarifies the scope for politically feasible reforms. A tax schedule T_0 is Pareto-efficient if there is no Pareto-improving reform. If it is Pareto-efficient, then for all y_a and y_b ,

$$y_b - y_a \geq \Delta_\tau^R(0, y_a, y_b) \geq 0 .$$

If we had instead $\Delta_\tau^R(0, y_a, y_b) < 0$, then a small reform (τ, y_a, y_b) with $\tau < 0$ would be Pareto-improving: all individuals would benefit from increased transfers and individuals with an income above y_a would, in addition, benefit from a tax cut. With $y_b - y_a < \Delta_\tau^R(0, y_a, y_b)$ a small reform (τ, y_a, y_b) with $\tau > 0$ would be Pareto-improving: all individuals would benefit from increased transfers. Individuals with an income above y_a would not benefit as much because of increased marginal tax rates. They would still be net beneficiaries because the increase of the tax burden was bounded by the increase of transfers. Hence, with $\Delta_\tau^R(0, y_a, y_b) < 0$ a reform that involves tax cuts is Pareto-improving and therefore politically feasible. With $y_b - y_a < \Delta_\tau^R(0, y_a, y_b)$ a reform that involves an increase of marginal tax rates is Pareto-improving and politically feasible. Under a Pareto-efficient tax system there is no scope for such reforms. We say that T_0 is an *interior Pareto-optimum* if, for all y_a and y_b ,

$$y_b - y_a > \Delta_\tau^R(0, y_a, y_b) > 0 .$$

Theorem 2 *Suppose that T_0 is an interior Pareto-optimum.*

- i) For $y_0 < \tilde{y}^0(w^M)$, there is a small reform (τ, y_a, y_b) with $y_a < y_0 < y_b$ and $\tau < 0$ that is politically feasible.*
- ii) For $y_0 > \tilde{y}^0(w^M)$, there is a small reform (τ, y_a, y_b) with $y_a < y_0 < y_b$ and $\tau > 0$ that is politically feasible.*

According to Theorem 2, if the status quo is an interior Pareto-optimum, reforms that involve a shift towards lower marginal tax rates for below median incomes and reforms that involve a shift towards higher marginal tax rates for above median incomes are politically feasible. With an interior Pareto-optimum, a lowering of marginal taxes for incomes between y_a and y_b comes with a loss of tax revenue. For individuals with incomes above y_b the reduction of their tax burden outweighs the loss of transfer income so that they benefit from such a reform. If y_b is smaller than the median income, this applies to all individuals with an income (weakly) above the median. Hence, the reform is politically feasible. By the same logic, an increase of marginal taxes for incomes between y_a and y_b generates additional tax revenue. If y_a is chosen so that $y_a \geq \tilde{y}^0(w^M)$, only individuals with above median income have to pay higher taxes with the consequence that all individuals with below median income, and hence a majority, benefit from the reform.

Theorem 2 allows us to identify politically feasible reforms. To see whether a given status quo in tax policy admits politically feasible reforms, we simply need to check whether the status quo is an interior Pareto-optimum. This in turn requires a characterization of Pareto bounds for tax rates. In the following, we will provide such a characterization. It takes the form of sufficient statistics formulas for the Pareto bounds that are associated with a given status quo in tax policy. Subsequently, we turn to politically feasible welfare improvements.

5.2 Pareto bounds for marginal tax rates

5.2.1 An upper bound for marginal tax rates

A reform (τ, y_a, y_b) with $\tau < 0$ is Pareto-improving if and only if

$$\Delta_\tau^R(\tau, y_a, y_b) \leq 0. \quad (5)$$

Proposition 4 below provides a characterization of the conditions under which such reforms exist. To be able to state the Proposition in a concise way, we define

$$\begin{aligned} \tilde{I}_0(\omega_0) &:= E [T'_0(\tilde{y}^0(\omega)) y_e^*(0, 0, \omega) \mid \omega \geq \omega_0] \\ &= \int_{\omega_0}^{\bar{\omega}} T'_0(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) \frac{f(\omega)}{1-F(\omega_0)} d\omega. \end{aligned} \quad (6)$$

Note that $y_e^*(0, 0, \omega) = 0$ and hence $\tilde{I}_0(\omega_0) = 0$ if there are no income effects. In the presence of income effects, $y_e^*(0, 0, \omega) < 0$ so that $\tilde{I}_0(\omega_0) < 0$ if marginal tax rates are

positive for types above ω_0 . Thus, $\tilde{I}_0(\omega_0)$ is a measure of income effects in the status quo. Also, let

$$D^R(\omega_0) := -\frac{1 - F(\omega_0)}{f(\omega_0)} \left(1 - \tilde{I}_0(\omega_0)\right) \frac{y_\omega^*(0, 0, \omega_0)}{y_\tau^*(0, 0, \omega_0)} \quad \text{and} \quad \mathcal{D}^R = D^R \circ \tilde{\omega}^0.$$

The function $D^R : \Omega \rightarrow \mathbb{R}$ is shaped by the hazard rate of the type distribution and by behavioral responses as reflected by the partial derivatives of the function y^* . The function $\mathcal{D}^R = D^R \circ \tilde{\omega}^0$ translates D^R into a function of earnings, rather than types. According to the following Proposition 4, \mathcal{D}^R is the upper Pareto bound for marginal tax rates.

Proposition 4

1. *Suppose that there is an income level y_0 so that $T'_0(y_0) < \mathcal{D}^R(y_0)$. Then there exists a revenue-increasing reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.*
2. *Suppose that there is an income level y_0 so that $T'_0(y_0) > \mathcal{D}^R(y_0)$. Then there exists a revenue-increasing reform (τ, y_a, y_b) with $\tau < 0$, and $y_a < y_0 < y_b$. This reform is also Pareto-improving.*

We provide an interpretation of \mathcal{D}^R as a sufficient statistics formula below. We first sketch its derivation. A detailed proof of Proposition 4 can be found in the Appendix.

For any reform in the (τ, y_a, y_b) -class, $\Delta^R(\tau, y_a, y_b)$ satisfies the fixed point equation

$$\Delta^R(\tau, y_a, y_b) = \int_{\underline{\omega}}^{\bar{\omega}} \{T_1(y^*(\Delta^R(\tau, y_a, y_b), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega .$$

Starting from this equation, we use the implicit function theorem and our analysis of behavioral responses to reforms in the (τ, y_a, y_b) -class in Appendix A to derive an expression for $\Delta^R_\tau(\tau, y_a, y_b)$. At this stage we do not yet invoke any assumption on τ , y_a and y_b . In particular, we do not assume a priori that τ is close to 0 or that y_b is close to y_a . We then evaluate this expression at $\tau = 0$ and obtain

$$\Delta^R_\tau(0, y_a, y_b) = \frac{1}{1 - I_0} \mathcal{R}(y_a, y_b),$$

where $I_0 := \tilde{I}_0(\underline{\omega})$ is our measure of income effects applied to the population at large and

$$\begin{aligned} \mathcal{R}(y_a, y_b) &= \int_{\omega_a}^{\omega_b} T'_0(y^*(0, 0, \omega)) y'_\tau(0, 0, \omega) f(\omega) d\omega \\ &\quad + \int_{\omega_a}^{\omega_b} \{y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad + (y_b - y_a)(1 - F(\omega_b)) \\ &\quad - (y_b - y_a) \int_{\omega_b}^{\bar{\omega}} T'_0(y^*(0, 0, \omega)) y'_e(0, 0, \omega) f(\omega) d\omega . \end{aligned}$$

The first entry in this expression is a measure of how people with incomes in (y_a, y_b) respond to the increased marginal tax rate and the second entry is the mechanical effect according to which these individuals pay more taxes for a given level of income. The third entry is the analogous mechanical effect for people with incomes above y_b and the

last entry gives their behavioral response due to the fact that they now operate on an income tax schedule with an intercept lower than c_0 .

We now exploit the assumption that there is no bunching under the initial schedule, so that $y^*(0, 0, \omega)$ is strictly increasing and hence invertible over $[\omega_a, \omega_b]$. We can therefore, without loss of generality, view a reform also as being defined by τ , ω_a and ω_b . With a slight abuse of notation, we can therefore write

$$\Delta_\tau^R(0, \omega_a, \omega_b) = \frac{1}{1 - I_0} \mathcal{R}(\omega_a, \omega_b),$$

where

$$\begin{aligned} \mathcal{R}(\omega_a, \omega_b) &= \int_{\omega_a}^{\omega_b} T'_0(y^*(0, 0, \omega)) y_\tau^*(0, 0, \omega) f(\omega) d\omega \\ &\quad + \int_{\omega_a}^{\omega_b} \{y^*(0, 0, \omega) - y^*(0, 0, \omega_a)\} f(\omega) d\omega \\ &\quad + (y^*(0, 0, \omega_b) - y^*(0, 0, \omega_a))(1 - F(\omega_b)) \\ &\quad - (y^*(0, 0, \omega_b) - y^*(0, 0, \omega_a)) \int_{\omega_b}^{\bar{\omega}} T'_0(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega. \end{aligned}$$

Note that $\Delta_\tau^R(0, \omega_a, \omega_a) = \frac{1}{1 - I_0} \mathcal{R}(\omega_a, \omega_a) = 0$. A small increase of marginal tax rates does not generate additional revenue if applied only to a null set of agents. However, if $\Delta_{\tau\omega_b}^R(0, \omega_a, \omega_a) = \frac{1}{1 - I_0} \mathcal{R}_{\omega_b}(\omega_a, \omega_a) > 0$, then $\Delta_\tau^R(0, \omega_a, \omega_b)$ turns positive, if starting from $\omega_a = \omega_b$, we marginally increase ω_b . Straightforward computations yield:

$$\mathcal{R}_{\omega_b}(\omega_a, \omega_a) = T'_0(y^*(0, 0, \omega_a)) y_\tau^*(0, 0, \omega_a) f(\omega_a) + (1 - \tilde{I}_0(\omega_a)) y_\omega^*(0, 0, \omega_a) (1 - F(\omega_a)).$$

Hence, if this expression is positive we can increase tax revenue by increasing marginal tax rates in a neighborhood of y_a . Using $y^*(0, 0, \omega_a) = y_a$, $\omega_a = \tilde{\omega}^0(y_a)$ and $y_\tau^*(0, 0, \omega_a) < 0$ (which is formally shown in the Appendix), the statement $\mathcal{R}_{\omega_b}(\omega_a, \omega_a) > 0$ is easily seen to be equivalent to $T'_0(y_a) < D^R(\omega_a) = \mathcal{D}^R(y_a)$, as claimed in the first part of Proposition 4 for $y_a = y_0$.

Sufficient statistics. Sufficient statistics approaches characterize the effects of tax policy by means of elasticities that describe behavioral responses. This route is also available here. To see this, define the elasticity of type ω_0 's earnings with respect to the net-of-tax rate $1 - T'(\cdot)$ in the status quo as

$$\tilde{\varepsilon}^0(\omega_0) \equiv \frac{1 - T'_0(\tilde{y}^0(\omega_0))}{\tilde{y}^0(\omega_0)} y_\tau^*(0, 0, \omega_0), \quad (7)$$

and the elasticity of earnings with respect to the skill index ω as

$$\tilde{\alpha}^0(\omega_0) \equiv \frac{\omega_0}{\tilde{y}^0(\omega_0)} y_\omega^*(0, 0, \omega_0). \quad (8)$$

We can also write

$$\tilde{I}(\omega_0) = E \left[T'_0(\tilde{y}^0(\omega)) \frac{\tilde{y}^0(\omega)}{c_0} \tilde{\eta}^0(\omega) \mid \omega \geq \omega_0 \right], \quad (9)$$

where $\tilde{\eta}^0(\omega) := \frac{c_0}{\tilde{y}^0(\omega)} y_e^*(0, 0, \omega)$ is the elasticity of type ω 's earnings with respect to exogenous income in the status quo.

Corollary 3 *Let*

$$\tilde{D}^R(\omega) := -\frac{1 - F(\omega)}{f(\omega)\omega} \left(1 - \tilde{I}_0(\omega)\right) \frac{\tilde{\alpha}^0(\omega)}{\tilde{\varepsilon}^0(\omega)} \quad \text{and} \quad \tilde{\mathcal{D}}^R = \tilde{D}^R \circ \tilde{\omega}^0.$$

1. *Suppose there is an income level y_0 so that $\frac{T'_0(y_0)}{1 - T'_0(y_0)} < \tilde{\mathcal{D}}^R(y_0)$. Then there exists a tax-revenue-increasing reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.*
2. *Suppose there is an income level y_0 so that $\frac{T'_0(y_0)}{1 - T'_0(y_0)} > \tilde{\mathcal{D}}^R(y_0)$. Then there exists a tax-revenue-increasing reform (τ, y_a, y_b) with $\tau < 0$, and $y_a < y_0 < y_b$.*

Corollary 3 is a particular version of what has become known as an *ABC*-formula, see Diamond (1998). The expression $\tilde{D}^R(\omega)$ is a product of an inverse hazard rate, usually referred to as *A*, an inverse elasticities term, *C*, and the third expression in the middle, *B*. Without income effects this middle term would simply be equal to 1. With income effects, however, we have to correct for the fact that a change of the intercept of the schedule $c_0 + y - T(y)$ affects the individuals' choices. In the presence of income effects, and with non-negative marginal tax rates in the status quo, $\tilde{I}_0(\omega_0) < 0$ because individuals become less eager to generate income if the intercept moves up. As a consequence, *B* exceeds 1 in the presence of income effects. Thus, everything else being equal, the right hand side becomes larger, so that there is more scope for revenue-increasing reforms if there are income effects. This effect can be illustrated by means of Figure 1. After the reform, the behavior of individuals with an income above y_b is as if they were facing a new schedule that differs from the old schedule only in the level of the intercept. The intercept is c_0 initially and $c_0 + \Delta^R - \tau(y_b - y_a) < c_0$ after the reform. Thus, for high income earners the intercept becomes smaller and they respond to this by increasing their earnings. These increased earnings translate into additional tax revenue. The term $-\tilde{I}_0(\omega_0) > 0$ takes account of this fiscal externality.

The following remark that we state without proof clarifies the implications of frequently invoked functional form assumptions for the sufficient statistics formula.

Remark 2 *If the utility function u takes the special form $u(c, y, \omega) = U\left(c, \frac{y}{\omega}\right)$ so that $\frac{y}{\omega}$ can be interpreted as labor supply in hours, then the ratio $\frac{\tilde{\alpha}^0(\omega)}{\tilde{\varepsilon}^0(\omega)}$ admits an interpretation in terms of the elasticity of hours worked with respect to the net wage rate $\tilde{\varepsilon}^0(\omega)$ so that $\frac{\tilde{\alpha}^0(\omega)}{\tilde{\varepsilon}^0(\omega)} = -\left(1 + \frac{1}{\tilde{\varepsilon}^0(\omega)}\right)$. If preferences are such that $u(c, y, \omega) = c - \left(\frac{y}{\omega}\right)^{1+\frac{1}{\epsilon}}$ for a fixed parameter ϵ , then, for all ω , $\frac{\tilde{\alpha}^0(\omega)}{\tilde{\varepsilon}^0(\omega)} = -\left(1 + \frac{1}{\epsilon}\right)$.*

An Example. We will repeatedly use a specific example of a status quo tax schedule for purposes of illustration. In this example, marginal tax rates are equal to 0 for incomes below an exemption threshold y^e , and constant for incomes above a top threshold y^t . In

between, marginal tax rates are a linearly increasing function of income. Consequently,

$$T'_0(y) = \begin{cases} 0, & \text{for } y \leq y^e, \\ \beta(y - y^e), & \text{for } y^e \leq y \leq y^t, \\ \beta(y^t - y^e), & \text{for } y \geq y^t; \end{cases} \quad (10)$$

where $\beta > 0$ determines how quickly marginal tax rates rise with income. Also note that, for $y^e \leq y \leq y^t$, the ratio $\frac{T'_0}{1-T'_0}$ is an increasing and convex function of income. Unless stated otherwise, the figures that follow are drawn under the assumption of a status quo schedule as in (10). It is also assumed that the distribution of types and the distribution of incomes are Pareto-distributions so that we can use Remark 1 to connect types and earnings. In addition, utility takes the form $u(c, y, \omega) = U(c, \frac{y}{\omega})$ so that, by Remark 2, the ratio $\frac{\tilde{\alpha}^0(\omega)}{\tilde{\varepsilon}^0(\omega)}$ is – via the relation $\frac{\tilde{\alpha}^0(\omega)}{\tilde{\varepsilon}^0(\omega)} = -\left(1 + \frac{1}{\tilde{\varepsilon}^0(\omega)}\right)$ – pinned down by the elasticity of hours worked with respect to the net wage. Finally, the figures are drawn under the assumption that the elasticities in $\{\tilde{\eta}^0(\omega)\}_{\omega \in \Omega}$ and $\{\tilde{\varepsilon}^0(\omega)\}_{\omega \in \Omega}$ are constant across types, i.e., that there exist numbers η and ε so that, for all ω , $\tilde{\eta}^0(\omega) = \eta$ and $\tilde{\varepsilon}^0(\omega) = \varepsilon$. The parameter choices are $\beta = 0.08$, $y^e = 5$ and $y^t = \frac{15}{2}$; $c_0 = 1$; $\omega_{min} = y_{min} = 1$, $\alpha = 1$, $a = 2$ and $b = 0.5$; $\eta = -0.02$ and $\varepsilon = 0.8$. The example only serves to illustrate our theoretical findings.

Remark 3 *The schedule \tilde{D}^R that allows us to identify revenue-increasing reforms does not generally coincide with a revenue-maximizing or Rawlsian income tax schedule. One reason is that \tilde{D}^R depends on the status quo schedule T_0 via the function \tilde{I}_0 . The status quo schedule will typically differ from the revenue-maximizing schedule. The dependence on the status quo schedule disappears if there are no income effects so that preferences can be described by a utility function that is quasi-linear in private goods consumption. One can show that, in this case, \tilde{D}^R coincides with the revenue-maximizing income tax schedule for all levels of income where the latter does not give rise to bunching.*

5.2.2 A lower bound for marginal tax rates

The following Proposition derives conditions under which reforms that yield higher marginal tax rates are Pareto-improving. It is the counterpart to Proposition 4 that characterizes Pareto-improving tax cuts.

Proposition 5 *Let*

$$D^P(\omega) := \frac{1}{f(\omega)} \left\{ \left(1 - \tilde{I}_0(\omega)\right) F(\omega) + \left(\tilde{I}_0(\omega) - I_0\right) \right\} \frac{y_\omega^*(0, 0, \omega)}{y_\tau^*(0, 0, \omega)} \quad \text{and} \quad \mathcal{D}^P := D^P \circ \tilde{\omega}^0.$$

Suppose that there is an income level y_0 such that $T'_0(y_0) < \mathcal{D}^P(y_0)$. Then there exists a Pareto-improving reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.

The function \mathcal{D}^P provides a lower bound for marginal tax rates. If marginal tax rates fall below this lower bound, then an increase of marginal tax rates is Pareto-improving.⁹ Note that $\mathcal{D}^P(y)$ is negative for low incomes.¹⁰ Thus, \mathcal{D}^P can be interpreted as a Pareto-bound on earnings subsidies. Such subsidies are, for instance, part of the earned income tax credit (EITC) in the US. If those subsidies imply marginal tax rates lower than those stipulated by \mathcal{D}^P , then a reduction of these subsidies is Pareto-improving.

The following Corollary is the counterpart to Corollary 3. It defines a sufficient statistic for Pareto-improving tax increases.

Corollary 4 *Let*

$$\tilde{D}^P(\omega) := \frac{1}{f(\omega)\omega} \left\{ (1 - \tilde{I}_0(\omega)) F(\omega) + (\tilde{I}_0(\omega) - I_0) \right\} \frac{\tilde{\alpha}^0(\omega)}{\tilde{\varepsilon}^0(\omega)} \quad \text{and} \quad \tilde{\mathcal{D}}^P := \tilde{D}^P \circ \tilde{\omega}^0.$$

Suppose that there is an income level y_0 such that $\frac{T'_0(y_0)}{1-T'_0(y_0)} < \tilde{\mathcal{D}}^P(y_0)$. Then there exists a Pareto-improving reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.

Figure 6 (left panel) represents \tilde{D}^R and \tilde{D}^P .

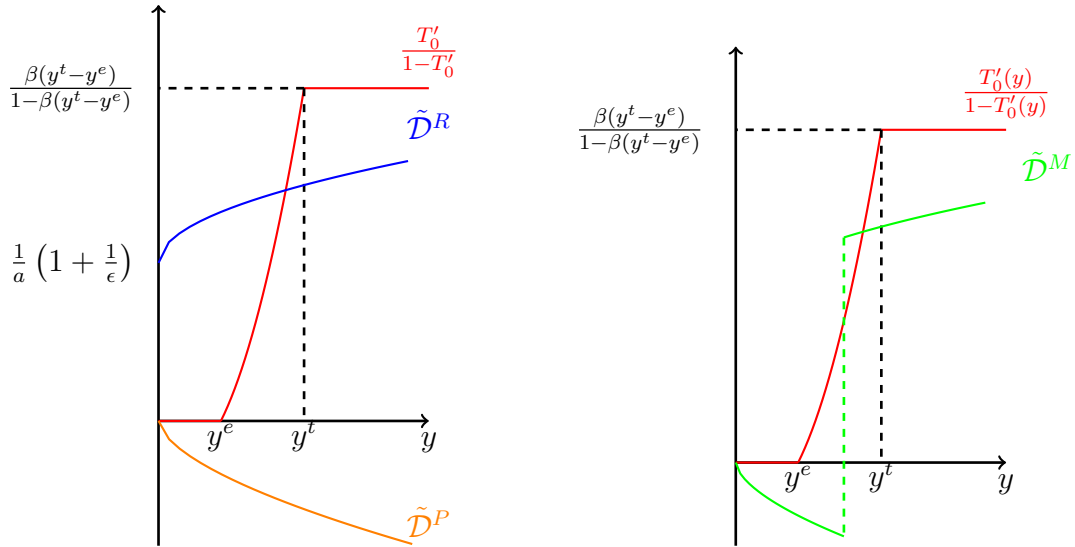


Figure 6: Sufficient statistics for Pareto-improving and Politically feasible reforms

⁹Under the assumption of quasi-linear in consumption preferences the \mathcal{D}^P -schedule admits an alternative interpretation as the solution of a taxation problem with the objective to maximize the utility of the most productive individuals subject to a resource constraint and an envelope condition which is necessary for incentive compatibility. Brett and Weymark (2017) refer to \tilde{D}^P as a maximax-schedule as it is derived from maximizing the utility of the most privileged individual.

¹⁰For low types, $\tilde{I}_0(\omega_0)$ is close to I_0 so that $\mathcal{D}^P(\omega_0)$ is approximately equal to $\frac{F(\omega_0)}{f(\omega_0)}(1 - I_0) \frac{y_{\omega}^*(0,0,\omega_0)}{y_{\tau}^*(0,0,\omega_0)} < 0$.

5.3 Politically feasible reforms

Propositions 4 and 5 characterize Pareto bounds for marginal tax rates. By Theorem 2, for a status quo such that marginal tax rates are between those bounds, tax increases for above median incomes and tax cuts for below median incomes are politically feasible. The following Proposition summarizes these findings.

Proposition 6 *Let*

$$\mathcal{D}^M(y) := \begin{cases} \mathcal{D}^P(y), & \text{if } y < \tilde{y}^0(\omega^M), \\ \mathcal{D}^R(y), & \text{if } y \geq \tilde{y}^0(\omega^M). \end{cases} \quad (11)$$

1. *Suppose there exists $y_0 \neq \tilde{y}^0(\omega^M)$ so that $T'_0(y_0) < \mathcal{D}^M(y_0)$. Then there is a politically feasible reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.*
2. *Suppose there exists $y_0 \neq \tilde{y}^0(\omega^M)$ so that $T'_0(y_0) > \mathcal{D}^M(y_0)$. Then there is a politically feasible reform (τ, y_a, y_b) with $\tau < 0$, and $y_a < y_0 < y_b$.*

Corollary 5 *Let*

$$\tilde{\mathcal{D}}^M(y) := \begin{cases} \tilde{\mathcal{D}}^P(y), & \text{if } y < \tilde{y}^0(\omega^M), \\ \tilde{\mathcal{D}}^R(y), & \text{if } y \geq \tilde{y}^0(\omega^M). \end{cases}$$

1. *Suppose that there is an income level y_0 so that $\frac{T'_0(y_0)}{1-T'_0(y_0)} < \tilde{\mathcal{D}}^M(y)$. Then there exists a politically feasible reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.*
2. *Suppose that there is an income level y_0 so that $\frac{T'_0(y_0)}{1-T'_0(y_0)} > \tilde{\mathcal{D}}^M(y)$. Then there exists a politically feasible reform (τ, y_a, y_b) with $\tau < 0$, and $y_a < y_0 < y_b$.*

Figure 6 (right panel) illustrates the relation between the sufficient statistics $\tilde{\mathcal{D}}^R$, $\tilde{\mathcal{D}}^P$ and $\tilde{\mathcal{D}}^M$. The figure shows that the schedule $\tilde{\mathcal{D}}^M$ is discontinuous at the median voter's type where it jumps from $\tilde{\mathcal{D}}^P$ to $\tilde{\mathcal{D}}^R$.

The discontinuous jump in Figure 6 offers an explanation for the observation that some real-world tax schedules are very steep in the middle.¹¹ The figure suggests that the median voter would appreciate any attempt to move quickly from $\tilde{\mathcal{D}}^P$ to $\tilde{\mathcal{D}}^R$ as one approaches median income. Any attempt to do so in a continuous fashion will imply a strong increase of marginal rates for incomes in a neighborhood of the median. This increase in marginal tax rates is the price to be paid for having marginal tax rates close to $\tilde{\mathcal{D}}^P$ for incomes below the median and for having marginal tax rates close to $\tilde{\mathcal{D}}^R$ for incomes above the median.

There is an analogy to Black (1948)'s theorem. According to this theorem, with single-peaked preferences over a one-dimension policy space, the majority rule is transitive. As

¹¹Germans refer to this as the "Mittelstandsbauch" (*middle class belly*) in the income tax schedule, for complementary evidence from the Netherlands see Zoutman, Jacobs and Jongen (2016).

a consequence, any sequence of pairwise majority votes will yield an outcome that is closer to the median's preferred policy than the status quo. Now consider our setup and suppose, for simplicity, that there are no income effects so that the Pareto bounds for marginal tax rates do not depend on the status quo. Any sequence of politically feasible tax reforms that affect only below median incomes will yield an outcome that is closer to the lower Pareto bound than the status quo. Any sequence of politically feasible tax reforms that affect only above median incomes will yield an outcome that is closer to the upper Pareto bound than the status quo. Such sequences will therefore make the discontinuity of the income tax schedule in a neighborhood of the median income more pronounced.

Remark 4 *The approach that yields a characterization of politically feasible reforms can be adapted so as to characterize reforms that are, say, appealing to the bottom x percent of the income distribution. The relevant auxiliary schedule for this purpose is*

$$\mathcal{D}^x(y) := \begin{cases} \mathcal{D}^P(y), & \text{if } y < \tilde{y}^0(\omega^x), \\ \mathcal{D}^R(y), & \text{if } y \geq \tilde{y}^0(\omega^x). \end{cases} \quad (12)$$

We can also define the corresponding sufficient statistic $\tilde{\mathcal{D}}^x$ in the obvious way. The discontinuity then shifts from the median to a different percentile of the income distribution.

5.4 Welfare-improving reforms

The following Proposition clarifies the conditions under which, for a given specification of welfare weights g , a small tax reform yields an increase in welfare. The following notation enables us to state the Proposition in a concise way. We define

$$\gamma_0 := \int_{\underline{\omega}}^{\bar{\omega}} g(\omega) \tilde{u}_c^0(\omega) f(\omega) d\omega,$$

where $\tilde{u}_c^0(\omega)$ is a shorthand for the marginal utility of consumption that a type ω individual realizes under the status quo, and

$$\Gamma_0(\omega') := \int_{\omega'}^{\bar{\omega}} g(\omega) \tilde{u}_c^0(\omega) \frac{f(\omega)}{1 - F(\omega')} d\omega.$$

Thus, γ_0 can be viewed as an average welfare weight in the status quo. It is obtained by multiplying each type's exogenous weight $g(\omega)$ with the marginal utility of consumption in the status quo $\tilde{u}_c^0(\omega)$, and then computing a population average. By contrast, $\Gamma_0(\omega')$ gives the average welfare weight of individuals with types above ω' . Note that $\gamma_0 = \Gamma_0(\underline{\omega})$ and that Γ_0 is a non-increasing function.

Proposition 7 *Let*

$$D_g^W(\omega) := -\frac{1 - F(\omega)}{f(\omega)} \Phi_0(\omega) \frac{y_\omega^*(0, 0, \omega)}{y_\tau^*(0, 0, \omega)} \quad \text{and} \quad \mathcal{D}_g^W = D_g^W \circ \tilde{\omega}^0,$$

where $\Phi_0(\omega) := 1 - \tilde{I}_0(\omega) - (1 - I_0) \frac{\Gamma_0(\omega)}{\gamma_0}$.

1. Suppose there is an income level y_0 so that $T'_0(y_0) < \mathcal{D}_g^W(y)$. Then there exists a welfare-increasing reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.
2. Suppose there is an income level y_0 so that $T'_0(y_0) > \mathcal{D}_g^W(y)$. Then there exists a welfare-increasing reform (τ, y_a, y_b) with $\tau < 0$, and $y_a < y_0 < y_b$.

The corresponding sufficient statistic \tilde{D}_g^W admits an easy interpretation if there are no income effects so that the utility function is quasi-linear in private goods consumption and if the costs of productive effort are iso-elastic. In this case, we have $I_0 = 0$, $\tilde{I}_0(\omega) = 0$, and $\tilde{u}_c^0(\omega) = 1$ for all ω . This implies, in particular, that $\gamma_0 = 1$, $\Gamma_0(\omega) = G(\omega)$, and $\Phi_0(\omega) = 1 - G(\omega)$, for all ω . Consequently,

$$\tilde{D}_g^W(\omega) = \frac{1 - F(\omega)}{f(\omega) \omega} (1 - G(\omega)) \left(1 + \frac{1}{\epsilon}\right),$$

where the right-hand side of this equation is the *ABC*-formula due to Diamond (1998). Again, in the absence of income effects, the sufficient statistic does not depend on the status quo schedule and coincides with the solution to a (relaxed) problem of welfare-maximizing income taxation.

If we take the status quo to be the laissez-faire situation with marginal tax rates of zero at all levels of income, we have

$$\tilde{D}_g^W(\omega) = -\frac{1 - F(\omega)}{f(\omega) \omega} \left(1 - \frac{\Gamma_0(\omega)}{\gamma_0}\right) \frac{\tilde{\alpha}^0(\omega)}{\tilde{\epsilon}^0(\omega)}.$$

Note that this expression will never turn negative, so that a welfare-maximizer would never want to move away from laissez-faire in the direction of earnings subsidies, or, equivalently negative marginal tax rates.

Figure 7 (left panel) relates the schedules \tilde{D}^P , \tilde{D}^R and \tilde{D}_g^W to each other. We draw \tilde{D}_g^W under the assumption that welfare weights take the form $g(\omega) = \frac{1}{1+\omega^2}$, for all ω . It shows the case of a status quo schedule that has inefficiently high tax rates at the top. At low levels of income, tax increases, while not mandated by Pareto-efficiency, would yield welfare improvements. For an intermediate range of incomes, status quo tax rates are within the Pareto bounds, but tax cuts would be welfare-improving.

5.5 Politically feasible welfare-improvements

Propositions 4 - 7 provide us with a characterization of the conditions under which a status quo tax policy admits reforms that are both politically feasible and welfare-improving. We summarize these findings in the following Proposition.

Proposition 8 *Given a status quo schedule T_0 and a specification of welfare weights g , there exists a politically feasible and welfare-increasing reform (τ, y_a, y_b) with $y_a < y_0 < y_b$, for $y_0 \neq \tilde{y}^0(\omega^M)$ if one of the following conditions is met:*

$$T'_0(y_0) < \mathcal{D}_g^W(y_0) \quad \text{and} \quad T'_0(y_0) < \mathcal{D}^M(y_0), \quad (13)$$

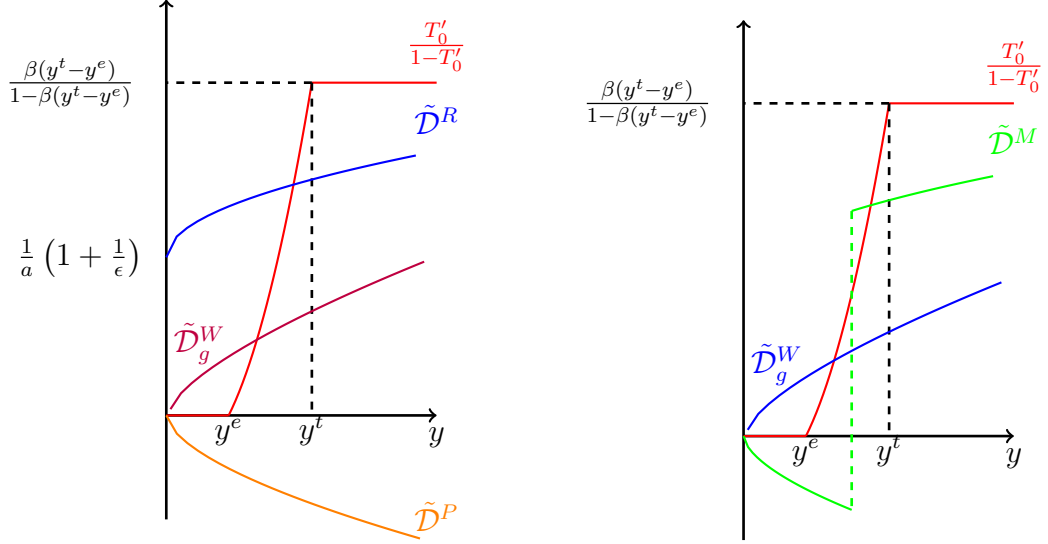


Figure 7: Sufficient statistics for Pareto-improving, Politically feasible, and Welfare-improving tax reforms

or

$$T'_0(y_0) > \mathcal{D}_g^W(y_0) \quad \text{and} \quad T'_0(y_0) > \mathcal{D}^M(y_0). \quad (14)$$

Proposition 8 states sufficient conditions for the existence of welfare-improving and politically feasible reforms. This raises the question of necessary conditions. Proposition 8 has been derived from focussing on “small” reforms, i.e., on small increases of marginal tax rates applied to a small range of incomes. The arguments in the proofs of Propositions 4 - 7 imply that if either condition (13) or condition (14) is violated at y_0 , then there is no small reform in the (τ, y_a, y_b) -class that is both welfare-improving and politically feasible.

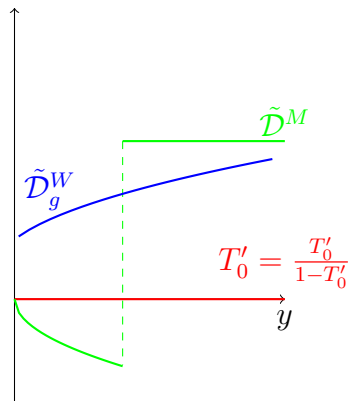


Figure 8: Sufficient statistics for politically feasible reforms and welfare-improving reforms with a laissez-faire status quo schedule

Figure 8 provides an illustration under the assumptions that the status quo schedule is the laissez-faire schedule.¹² The figure shows that for incomes above the median, an increase in marginal tax rates is both welfare-improving and politically feasible. This holds for any welfare function under which the function g is strictly decreasing.¹³ By contrast, for incomes below the median, there is no reform that is both politically feasible and welfare-improving: welfare improvements require to raise marginal taxes relative to the laissez-faire ones, whereas political feasibility requires to introduce negative marginal tax rates.

For Figure 7 (right panel), the status quo tax schedule is, again, as in (10). In this example, for high incomes, tax rates are inefficiently high so that tax cuts are both politically feasible and welfare-improving. There is a range of incomes above the median income where tax cuts are not mandated by Pareto-efficiency. In this region, tax increases are therefore politically feasible. They are, however, not desirable for the given welfare function. For a range of incomes below the median income, tax cuts are politically feasible and welfare-improving. For low incomes, tax cuts are politically feasible, but welfare-damaging.

Our analysis in this section provides a diagnosis system that can be used to identify reform options that are associated with a given income tax system. In particular, we can check whether there is scope for revenue increases, Pareto-improvements, welfare improvements and, moreover, for reforms that are politically feasible in the sense that they make a majority of individuals better off.

The analysis suggests that existing tax schedules might be viewed as resulting from a compromise between concerns for welfare-maximization on the one hand, and concerns for political support on the other. If the maximization of political support was the only force in the determination of tax policy, we would expect to see tax rates close to the revenue-maximizing rate \tilde{D}^R for incomes above the median and negative rates close to \tilde{D}^P for incomes below the median. Concerns for welfare dampen these effects. A welfare-maximizing approach will generally yield higher marginal tax rates for incomes below the median and lower marginal tax rates for incomes above the median.

Our analysis also raises a question. Diamond (1998) and Saez (2001) have argued that, for plausible specifications of welfare weights, existing tax schedules have marginal tax rates for high incomes that are too low. Our analysis would suggest that an increase of these tax rates would not only be welfare-improving but also politically feasible. Why don't we see more reforms that involve higher tax rates for the rich? Proposition 1 provides a possible answer to this question: reforms that do not belong to the (τ, y_a, y_b) -class but instead involve tax cuts that are larger for richer taxpayers may as well prove to be politically feasible. For such reforms political feasibility requires that the median

¹²A reform of a laissez-faire status quo schedule corresponds to the introduction of an income tax schedule as studied in Aidt and Jensen (2009).

¹³If $g(\omega) = 1$, for all ω , then such tax increases are politically feasible, but not welfare improving.

voter is included in the set of those who benefit from the tax cuts.

6 Extensions

In this section we show that the median voter theorem for small monotonic reforms (Theorem 1) applies to models with more than one source of heterogeneity among individuals. Again, we show that a small tax reform is preferred by a majority of taxpayers if and only if it is preferred by the taxpayer with median income. Throughout, we stick to the assumption that individuals differ in their productive abilities ω . We introduce a second consumption good and the possibility of heterogeneity in preferences over consumption goods in Section 6.1. We use this framework to discuss whether the introduction of distortionary taxes on savings is politically feasible. In Section 6.2 we consider fixed costs of labor market participation as an additional source of heterogeneity.¹⁴ In Section 6.3 we assume that individuals differ in their valuation of increased public spending.¹⁵ Finally, in Section 6.4, individuals differ by how much of their income is due to luck as in Alesina and Angeletos (2005).

6.1 Political support for taxes on savings

We now suppose that there are two consumption goods. We refer to them as food and savings, respectively. An individual's budget constraint now reads as

$$c_f + c_s + T_{0s}(c_s) + \tau_s h_s(c_s) \leq c_0 + y - T_0(y) - \tau h(y) . \quad (15)$$

The variables on the right-hand side of the budget constraint have been defined before. On the left-hand side, c_f denotes food consumption and c_s savings. In the status quo savings are taxed according to a possibly non-linear savings-tax function T_{0s} . A reform replaces both the status quo income tax schedule T_0 by $T_1 = T_0 + \tau h$ and the status quo savings tax schedule T_{0s} by $T_{1s} = T_{0s} + \tau_s h_s$. We maintain the assumption that the functions h and h_s are non-decreasing and focus on revenue neutral reforms so that either $\tau > 0$ and $\tau_s < 0$ or $\tau < 0$ and $\tau_s > 0$.

Preferences of individuals are given by a utility function $u(v(c_f, c_s, \beta), y, \omega)$, where v is a subutility function that assigns consumption utility to any consumption bundle (c_f, c_s) . The marginal rate of substitution between food and savings depends on a parameter β . We do not assume a priori that β is the same for all individuals. Under this assumption, however, the utility function u has the properties under which an efficient tax system does not involve distortionary commodity taxes, see Atkinson-Stiglitz (1976), or Laroque (2005) for a more elementary proof. Distortionary taxes on savings are then undesirable from a welfare-perspective.

¹⁴See Saez (2002), Choné and Laroque (2011), and Jacquet, Lehmann and Van der Linden (2013).

¹⁵See Boadway and Keen (1993), Hellwig (2004), Bierbrauer and Sahm (2010), Bierbrauer (2014), or Weinzierl (forthcoming).

Individuals choose c_f , c_s and y to maximize utility subject to the budget constraint above. We denote the utility maximizing choices by $c_f^*(\tau_s, \tau, \beta, \omega)$, $c_s^*(\tau_s, \tau, \beta, \omega)$ and $y^*(\tau_s, \tau, \beta, \omega)$ and the corresponding level of indirect utility by $V(\tau_s, \tau, \beta, \omega)$. The slope of an indifference curve in a τ - τ_s diagram determines the individuals' willingness to accept higher savings taxes in return for lower taxes on current earnings. The following Lemma provides a characterization of this marginal rate of substitution in a neighborhood of the status quo. Let

$$s(\tau, \tau^s, \beta, \omega) = -\frac{V_\tau(\tau_s, \tau, \beta, \omega)}{V_{\tau_s}(\tau_s, \tau, \beta, \omega)}$$

be the slope of an individual's indifference curve in a τ - τ_s diagram. The slope in the status quo is denoted by $s^0(\omega, \beta)$. We denote the individual's food consumption, savings and earnings in the status quo by $\tilde{c}_f^0(\omega, \beta)$, $\tilde{c}_s^0(\omega, \beta)$ and $\tilde{y}^0(\omega, \beta)$, respectively.

Lemma 4 *In the status quo the slope of a type (ω, β) -individual's indifference curve in a τ - τ_s diagram is given by*

$$s^0(\omega, \beta) = -\frac{h(\tilde{y}^0(\omega, \beta))}{h_s(\tilde{c}_s^0(\omega, \beta))}.$$

The proof of Lemma 4 can be found in the Appendix. The Lemma provides a generalization of Roy's identity that is useful for an analysis of non-linear tax systems. As is well known, with linear tax systems, the marginal effect of, say, an increased savings tax on indirect utility is equal to $-\lambda^* c_s^*(\cdot)$, where λ^* is the multiplier on the individual's budget constraint, also referred to as the marginal utility of income. Analogously, the increase of a linear income tax affects indirect utility via $-\lambda^* y^*(\cdot)$ so that the slope of an indifference curve in a τ_s - τ -diagram would be equal to the earnings-savings-ratio $-\frac{y^*(\cdot)}{c_s^*(\cdot)}$. Allowing for non-linear tax systems and non-linear perturbations implies that the simple earnings-savings-ratio is replaced by $-\frac{h(y^*(\cdot))}{h_s(c_s^*(\cdot))}$.

Consider a reform that involves an increase in the savings tax rate $d\tau_s > 0$ and a reduction of taxes on income $d\tau < 0$. We say that a type (ω, β) -individual strictly prefers a small reform with increased savings taxes over the status quo if

$$V_{\tau_s}(0, 0, \beta, \omega) d\tau_s + V_\tau(0, 0, \beta, \omega) d\tau > 0,$$

or, equivalently, if

$$\frac{d\tau_s}{d\tau} > s^0(\omega, \beta) = -\frac{h(\tilde{y}^0(\omega, \beta))}{h_s(\tilde{c}_s^0(\omega, \beta))}. \quad (16)$$

Since h_s is an increasing function, this condition is, ceteris paribus, easier to satisfy if the individual has little savings in the status quo. The ratio $\frac{d\tau_s}{d\tau}$ on the left-hand side of inequality (16) follows from the behavioral responses of individuals to a small revenue-neutral tax reform. Let $\Delta^{R_s}(\tau_s, \tau)$ be the change of revenue from savings taxes

and $\Delta^R(\tau_s, \tau)$ the change of revenue from income taxation due to the reform. Revenue-neutrality requires that

$$\Delta_{\tau_s}^{R_s}(\tau_s, \tau) d\tau_s + \Delta_{\tau}^{R_s}(\tau_s, \tau) d\tau + \Delta_{\tau_s}^R(\tau_s, \tau) d\tau_s + \Delta_{\tau}^R(\tau_s, \tau) d\tau = 0,$$

or, equivalently, that

$$\frac{d\tau_s}{d\tau} = - \frac{\Delta_{\tau}^R(\tau_s, \tau) + \Delta_{\tau}^{R_s}(\tau_s, \tau)}{\Delta_{\tau_s}^{R_s}(\tau_s, \tau) + \Delta_{\tau_s}^R(\tau_s, \tau)},$$

which has to be evaluated for $(\tau_s, \tau) = (0, 0)$. We simply assume that, for the reforms that we consider, this expression is well-defined and takes a finite negative value.

Different types will typically differ in their generalized earnings-savings-ratio $s^0(\omega, \beta)$ and we can order types according to this one-dimensional index. Let $(\omega, \beta)^{0M}$ be the type with the median value of $s^0(\omega, \beta)$. The following proposition extends Theorem 1. It asserts that a small reform is politically feasible if and only if it is supported by the median type $(\omega, \beta)^{0M}$.

Proposition 9 *For a given status quo tax policy and a given pair of non-decreasing functions h and h_s , the following statements are equivalent:*

1. *Type $(\omega, \beta)^{0M}$ prefers a small reform with increased savings taxes over the status quo.*
2. *There is a majority of individuals who prefer a small reform with increased savings taxes over the status quo.*

As Theorem 1, Proposition 9 exploits the observation that individuals can be ordered according to a one-dimensional statistic that pins down whether or not they benefit from a tax reform. This makes it possible to prove a median-voter theorem for reforms that remain in a neighborhood of the status quo. There is also a difference to Theorem 1. With only one-dimensional heterogeneity, there is a monotonic relation between types and earnings so that the identity of the type with median income does not depend on the status quo. Whatever the tax system, the person with the median income is the person with the median type ω^M . Here, by contrast, we allow for heterogeneity both in productive abilities and in preferences over consumption goods. The type with the median value of the generalized earnings-savings-ratio $s^0(\omega, \beta)$ will then typically depend on the status quo tax system. This does not pose a problem if we focus on small reforms. In this case, preferences over reforms follow from the generalized earnings-savings-ratios in the status quo, and a small reform is preferred by a majority of individuals if and only if it is preferred by the individual with the median ratio.

6.2 Fixed costs of labor market participation

With fixed costs of labor market participation individuals derive utility $u(c - \theta \mathbf{1}_{y>0}, y, \omega)$ from a (c, y) -pair. Fixed costs θ absorb some of the individual's after-tax income if the individual becomes active on the labor market, e.g. because of additional child care expenses. As before, there is an initial status quo tax schedule under which earnings are transformed into after-tax income according to the schedule C_0 with $C_0(y) = c_0 + y - T_0(y)$. After a reform, the schedule is

$$C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau h(y) ,$$

where h is a non-decreasing function of y . We denote by $y^*(\Delta^R, \tau, \omega, \theta)$ the solution to

$$\max_y u(C_1(y) - \theta \mathbf{1}_{y>0}, y, \omega),$$

and the corresponding level of indirect utility by $V(\Delta^R, \tau, \omega, \theta)$. We proceed analogously for other variables: what has been a function of ω in previous sections is now a function of ω and θ .

For a given function h , the marginal gain that is realized by an individual with type (ω, θ) if the tax rate τ is increased, is given by the following analogue to equation (4),

$$\Delta_\tau^V(\omega, \theta \mid \tau, h) = \tilde{u}_c^1(\omega, \theta) (\Delta_\tau^R(\tau, h) - h(\tilde{y}^1(\omega, \theta))) , \quad (17)$$

where $\tilde{u}_c^1(\omega, \theta)$ is the marginal utility of consumption realized by a type (ω, θ) -individual after the reform, and $\tilde{y}^1(\omega, \theta)$ are the individual's post-reform earnings. At $\tau = 0$, we can also write

$$\Delta_\tau^V(\omega, \theta \mid 0, h) = \tilde{u}_c^0(\omega, \theta) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega, \theta))) , \quad (18)$$

where $\tilde{u}_c^0(\omega, \theta)$ and $\tilde{y}^0(\omega, \theta)$ are, respectively, marginal utility of consumption and earnings in the status quo.

For a given status quo tax policy and a given function h we say that type (ω, θ) strictly prefers a small tax reform over the status quo if $\Delta_\tau^V(\omega, \theta \mid 0, h) > 0$. The status quo median voter strictly prefers a small reform if $\Delta_\tau^V((\omega, \theta)^{0M} \mid 0, h) > 0$, where y^{0M} is the median of the distribution of earnings in the status quo and $(\omega, \theta)^{0M}$ is the corresponding type; i.e. $\tilde{y}^0((\omega, \theta)^{0M}) = y^{0M}$.

Proposition 10 *For a given status quo tax policy and a monotonic function h , the following statements are equivalent:*

1. *Type $(\omega, \theta)^{0M}$ prefers a small reform over the status quo.*
2. *There is a majority of individuals who prefer a small reform over the status quo.*

Proposition 10 exploits that the slope of a type (ω, θ) individual's indifference curve through a point (τ, Δ^R) ,

$$s(\tau, \Delta^R, \omega, \theta) = h(y^*(\Delta^R, \tau, \omega, \theta)) .$$

is a function of the individual's income. As in the basic Mirrleesian setup, the interpretation is that individuals with a higher income are more difficult to convince that a reform that involves tax increases ($\tau > 0$) is worthwhile. A difference to the Mirrleesian setup is, however, that there is no monotonic relation between types and earnings. In the presence of income effects, and for a given level of ω , y^* will increase in θ as long as θ is below a threshold $\hat{\theta}(\omega)$ and be equal to 0 for θ above the threshold. This also implies that there is no longer a fixed type whose income is equal to the median income whatever the tax schedule. As in Proposition 9, this does not pose a problem if we focus on small reforms, i.e. on small deviations from $(\tau, \Delta^R) = (0, 0)$. In this case, preferences over reforms follow from the relation between types and earnings in the status quo, and a small reform is preferred by a majority of individuals if and only if it is preferred by the individual with the median level of income in the status quo.

6.3 Public-goods preferences

Suppose that the change in revenue Δ^R is used to increase or decrease spending on publicly provided goods. The post-reform consumption schedule is then given by

$$C_1(y) = c_0 + y - T_0(y) - \tau h(y) ,$$

We assume that individuals differ with respect to their public-goods preferences. Now the parameter θ is a measure of an individual's willingness to give up private goods consumption in exchange for more public goods. More specifically, we assume that individual utility is

$$u(\theta(R^0 + \Delta^R) + C_1(y), y, \omega) ,$$

where R^0 is spending on publicly provided goods in the status quo. Again, we denote by $y^*(\Delta^R, \tau, \omega, \theta)$ the solution to

$$\max_y u(\theta(R^0 + \Delta^R) + C_1(y), y, \omega)$$

and indirect utility by $V(\Delta^R, \tau, \omega, \theta)$. By the envelope theorem, the slope of a type (ω, θ) individual's indifference curve through point (τ, Δ^R) is now given by

$$s(\tau, \Delta^R, \omega, \theta) = \frac{h(y^*(\Delta^R, \tau, \omega, \theta))}{\theta} .$$

This marginal rate of substitution gives the increase in public-goods provision that an individual requires as a compensation for an increase of marginal tax rates. *Ceteris paribus*, individuals with a lower income and individuals with a higher public-goods

preference require less of a compensation, i.e. they have a higher willingness to pay higher taxes for increased public-goods provision. If we focus on small reforms we observe, again, that if a type (ω, θ) -individual benefits from a small tax-increase, then the same is true for any type (ω', θ') with

$$\frac{h(\tilde{y}^0(\omega, \theta))}{\theta} \geq \frac{h(\tilde{y}^0(\omega', \theta'))}{\theta'}.$$

By the arguments in the proof of Proposition 10, a small reform with $\tau > 0$ is preferred by a majority of individuals if and only if

$$\left(\frac{h(\tilde{y}^0(\omega, \theta))}{\theta} \right)^{0M} < \frac{d\Delta^R}{d\tau},$$

where $\left(\frac{h(\tilde{y}^0(\omega, \theta))}{\theta} \right)^{0M}$ is the median willingness to pay higher taxes for increased public spending in the status quo.

6.4 Fairness and politically feasible reforms

We analyze politically feasible reforms of non-linear tax systems for a setup in which individual incomes can be due to luck or effort and in which preferences over reforms include a motive to tax income that is due to luck more heavily than income that is due to effort. In modeling such preferences we adopt the framework of Alesina and Angeletos (2005). Alesina and Angeletos focus, however, on linear tax systems.

There are two periods. When young individuals choose a level of human capital k . When old individuals choose productive effort or labor supply l . Pre-tax income is determined by

$$y = \pi(l, k) + \eta,$$

where π is a production function that is increasing in both arguments and η is a random source of income, also referred to as luck. An individual's life-time utility is written as $u(c, l, k, \omega)$. Utility is increasing in the first argument. It is decreasing in the second and third argument to capture the effort costs of labor supply and human capital investments, respectively. Effort costs are decreasing in ω . More formally, lower types have steeper indifference curves both in a (c, l) -space and in a (c, k) -space. We consider reforms that lead to a consumption schedule

$$C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau h(y).$$

We assume that individuals first observe how lucky they are and then choose how hard they work, i.e. given a realization of η and given the predetermined level of k , individuals choose l so as to maximize

$$u(C_1(\pi(l, k) + \eta), l, k, \omega).$$

We denote the solution to this problem by $l^*(\Delta^R, \tau, \omega, \eta, k)$. Indirect utility is denoted by $V(\Delta^R, \tau, \omega, \eta, k)$. As of $t = 1$, there is multi-dimensional heterogeneity among individuals: they differ in their type ω , in their realization of luck η and possibly also in their human capital k .

In Alesina and Angeletos (2005) preferences over reforms have a selfish and fairness component. The indirect utility function V shapes the individuals' selfish preferences over reforms, for predetermined levels of human capital. The analysis of these selfish preferences can proceed along similar lines as the extension that considered fixed costs of labor market participation. Selfish preferences over small reforms follow from the relation between types and earnings in the status quo, and a small reform makes a majority better off if and only if it is beneficial for the individual with the median level of income in the status quo. More formally, let $\tilde{y}^0(\omega, \eta, k) := y^*(0, 0, \omega, \eta, k)$ be a shorthand for the earnings of a type (ω, η, k) -individual in the status quo and recall that the sign of

$$s(0, 0, \omega, \eta, k) = h(\tilde{y}^0(\omega, \eta, k))$$

determines whether an individual benefits from a small tax reform. Specifically, suppose that h is a non-decreasing function and denote by y^{0M} the median level of income in the status quo and by $(\omega, \eta, k)^{0M}$ the corresponding type. A majority of individuals is – according to their selfish preferences – made better off by reform if and only if the median voter benefits from the reform,

$$s^0((\omega, \eta, k)^{0M}) = h(\tilde{y}^{0M}) < \frac{d\Delta^R}{d\tau} .$$

In their formalization of social preferences, Alesina and Angeletos (2005) view $\pi(l, k)$ as a reference income. It is the part of income that is due to effort as opposed to luck. A tax reform affects the share of $y = \pi(l, k) + \eta$ that individuals can keep for themselves. After the reform, the difference between disposable income and the reference income is given by¹⁶

$$C_1(y) - \pi(l, k) = \eta - T_0(\pi(l, k) + \eta) - \tau h(\pi(l, k) + \eta) .$$

A social preferences for fair taxes is then equated with a desire to minimize the variance of $\eta - T_0(\pi(l, k) + \eta) - \tau h(\pi(l, k) + \eta)$ taking into account that k and l are endogenous variables.¹⁷ Denote this variance henceforth by $\Sigma(\Delta^R, \tau)$. Any one individual is assumed to evaluate a tax reform according to

$$V(\Delta^R, \tau, \omega, \eta, k) - \rho \Sigma(\Delta^R, \tau) ,$$

¹⁶The analysis in Alesina and Angeletos (2005) looks at a special case of this. They focus on a status quo equal to the laissez-faire schedule so that $T_0(y) = 0$, for all y , and a reform that introduces a linear tax schedule, i.e. $h(y) = y$, for all y . Under these assumptions, we have $\eta - T_0(\pi(l, k) + \eta) - \tau h(\pi(l, k) + \eta) = (1 - \tau)\eta + \tau\pi(l, k)$.

¹⁷Human capital investment is a function of effort costs ω and the expectations (Δ^{Re}, τ^e) of the young on the tax reforms that will be adopted when they are old.

where ρ is the weight on fairness considerations which is assumed to be the same for all individuals. Therefore, heterogeneity in preferences over reforms is entirely due to heterogeneity in selfish preferences. Consequently, the finding that a small reform is preferred by a majority of taxpayers if and only if it is preferred by the voter with median income in the status quo is not affected by the inclusion of a demand for fair taxes.

7 Concluding remarks

This paper develops a framework for an analysis of tax reforms. This framework can be applied to any given income tax system. It makes it possible to identify reforms that are politically feasible in the sense that they would be supported by a majority of taxpayers or to identify welfare-improving reforms. One can also study the intersection of politically feasible and welfare-improving reforms. If this set is empty, the status quo is constrained efficient because the scope for politically feasible welfare-improvements has been exhausted.

With non-linear tax systems, the policy-space is multi-dimensional with the implication that political economy forces are difficult to characterize. A main result in our paper is that this difficulty can be overcome by looking at tax reforms that are monotonic such that the change in tax payments is a monotonic function of income. We show that, with such a policy space, a reform is preferred by a majority of taxpayers if and only if it is preferred by the taxpayer with median income. This implies that tax decreases for incomes below the median and tax increases for incomes above the median are politically feasible, provided that tax rates stay within Pareto bounds.

Our analysis identifies, for each level of income, Pareto-bounds for marginal tax rates. It also identifies reforms that would be preferred by a majority of voters. Moreover, we derive sufficient statistics that make it possible to bring this framework to the data. This allows to diagnose what types of reforms are possible under a given status quo tax policy. Future research might use this framework to complement existing studies on the history of income taxation. For instance, Scheve and Stasavage (2016) study whether tax systems have become more progressive in response to increases in inequality or in response to extensions of the franchise. Their analysis compares tax policies that have been adopted at different points in time, or by countries with different institutions. It does not include an analysis of the reforms that appear to have been politically feasible or welfare-improving in a given year for a given country and a given status quo tax schedule. The framework that is developed in this paper lends itself to such an analysis.

Finally, we develop our analysis mainly in the context of a Mirrleesian model of income taxation in which individuals differ in their costs of effort. A similar analysis would be possible for any model of taxation in which a version of the Spence-Mirrlees single crossing property holds. We also show that our analysis of small monotonic reforms ex-

tends to models with multi-dimensional heterogeneity of individuals, such as models with variable and fixed costs of labor market participation, models that include heterogeneity in preferences over public goods, or models that include an investment in human capital.

References

- Acemoglu, D., M. Golosov, and A. Tsyvinski**, “Political Economy of Mechanisms,” *Econometrica*, 2008, *76*, 619–641.
- , —, and —, “Dynamic Mirrlees Taxation under Political Economy Constraints,” *Review of Economic Studies*, 2010, *77*, 841–881.
- , **S. Naidu, P. Restrepo, and J.A. Robinson**, “Democracy, Redistribution, and Inequality,” *Handbook of Income Distribution*, 2015, pp. 1885–1966.
- Aidt, T.S. and P.S. Jensen**, “The taxman tools up: An event history study of the introduction of the personal income tax,” *Journal of Public Economics*, 2009, *93* (1), 160–175.
- Alesina, A. and G.-M. Angeletos**, “Fairness and Redistribution,” *American Economic Review*, 2005, *95* (4), 960–980.
- , **S. Stantcheva, and E. Teso**, “Intergenerational Mobility and Support for Redistribution,” *NBER Working Paper 23027*, 2017.
- Atkinson, A. and J. Stiglitz**, “The Design of Tax Structure: Direct versus Indirect Taxation,” *Journal of Public Economics*, 1976, *1*, 55–75.
- Bargain, O., M. Dolls, D. Neumann, A. Peichl, and S. Sieglösch**, “Tax-Benefit Systems in Europe and the US: Between Equity and Efficiency,” *IZA Discussion Papers 5440*, 2011.
- Baron, D. and J. Ferejohn**, “Bargaining in Legislatures,” *American Political Science Review*, 1989, *83*, 1181–1206.
- Barro, R.**, “The Control of Politicians: An economic model,” *Public Choice*, 1973, *14*, 19–42.
- Bastani, S. and J. Lundberg**, “Political preferences for redistribution in Sweden,” *Working Paper*, 2016.
- Battaglini, M. and S. Coate**, “A Dynamic Theory of Public Spending, Taxation and Debt,” *American Economic Review*, 2008, *98*, 201–236.
- Bénabou, R. and E.A. Ok**, “Social Mobility and the Demand for Redistribution: The Poup Hypothesis,” *Quarterly Journal of Economics*, 2001, *116* (2), 447–487.

- and **J. Tirole**, “Belief in a just world and redistributive politics,” *Quarterly Journal of Economics*, 2006, *121* (2), 699–746.
- Bierbrauer, F.J.**, “Optimal Tax and Expenditure Policy with Aggregate Uncertainty,” *American Economic Journal: Microeconomics*, 2014, *6* (1), 205–57.
- and **M. Sahm**, “Optimal democratic mechanisms for taxation and public good provision,” *Journal of Public Economics*, 2010, *94* (78), 453 – 466.
- and **P.C. Boyer**, “Efficiency, Welfare, and Political Competition,” *Quarterly Journal of Economics*, 2016, *131* (1), 461–518.
- Black, D.**, “On the Rationale of Group Decision-making,” *Journal of Political Economy*, 1948, *56* (1), 23–34.
- Blundell, R., M. Brewer, P. Haan, and A. Shephard**, “Optimal income taxation of lone mothers: an empirical comparison of the UK and Germany,” *Economic Journal*, 2009, *119*, F101–F121.
- Boadway, R. and M. Keen**, “Public Goods, Self-Selection and Optimal Income Taxation,” *International Economic Review*, 1993, *34*, 463–478.
- Bourguignon, F. and A. Spadaro**, “Tax-benefit Revealed Social Preferences,” *Journal of Economic Inequality*, 2012, *10* (1), 75–108.
- Brett, C. and J.A. Weymark**, “Voting over selfishly optimal nonlinear income tax schedules with a minimum-utility constraint,” *Journal of Mathematical Economics*, 2016, *67*, 18–31.
- and — , “Voting over selfishly optimal nonlinear income tax schedules,” *Games and Economic Behavior*, 2017, *101*, 172–188.
- Choné, P. and G. Laroque**, “Optimal taxation in the extensive model,” *Journal of Economic Theory*, 2011, *146* (2), 425–453.
- Christiansen, V. and E. Jansen**, “Implicit Social Preferences in the Norwegian System of Indirect Taxation,” *Journal of Public Economics*, 1978, *10* (2), 217–245.
- Diamond, P.A.**, “Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates,” *American Economic Review*, 1998, *88*, 83–95.
- and **E. Saez**, “The Case for a Progressive Tax: From Basic Research to Policy Recommendations,” *Journal of Economic Perspectives*, 2011, *25* (4), 165–90.
- Drazen, A.**, *Political Economy in Macroeconomics*, Princeton University Press, 2000.

- Farhi, E., C. Sleet, I. Werning, and S. Yeltekin**, “Non-linear Capital Taxation Without Commitment,” *Review of Economic Studies*, 2012, 79 (4), 1469–1493.
- Feldstein, Martin**, “On the theory of tax reform,” *Journal of Public Economics*, 1976, 6 (12), 77 – 104.
- Ferejohn, J.**, “Incumbent Performance and Electoral Control,” *Public Choice*, 1986, 50, 5–25.
- Gans, J.S. and M. Smart**, “Majority voting with single-crossing preferences,” *Journal of Public Economics*, 1996, 59 (2), 219 – 237.
- Golosov, M., A. Tsyvinski, and N. Werquin**, “A Variational Approach to the Analysis of Tax Systems,” Working Paper 20780, National Bureau of Economic Research December 2014.
- Guesnerie, R.**, *A Contribution to the Pure Theory of Taxation*, Cambridge University Press, 1995.
- Heathcote, J., K. Storesletten, and G.L. Violante**, “Optimal Tax Progressivity: An Analytical Framework,” *Quarterly Journal of Economics*, forthcoming.
- Hellwig, M.F.**, “Optimal income taxation, public-goods provision and public-sector pricing: A contribution to the foundations of public economics,” *MPI Collective Goods Preprint*, 2004, 2004/14.
- , “A Contribution to the Theory of Optimal Utilitarian Income Taxation,” *Journal of Public Economics*, 2007, 91, 1449–1477.
- Hendren, N.**, “The Inequality Deflator: Interpersonal Comparisons without a Social Welfare Function,” *NBER Working Paper 20351*, 2014.
- Ilzetzki, E.**, “A positive theory of tax reform,” *Mimeo, London School of Economics*, 2015.
- Jacquet, L. and E. Lehmann**, “Optimal Taxation with Heterogeneous Skills and Elasticities: Structural and Sufficient Statistics Approaches,” *THEMA Working Paper 2016/04*, 2016.
- , —, and **B. Van der Linden**, “Optimal redistributive taxation with both extensive and intensive responses,” *Journal of Economic Theory*, 2013, 148 (5), 1770 – 1805.
- Kleven, H.J.**, “Bunching,” *Annual Review of Economics*, 2016, 8, 435–464.
- Laroque, G.**, “Indirect taxation is superfluous under separability and taste homogeneity: a simple proof,” *Economic Letters*, 2005, 87, 141–144.

- Lindbeck, A. and J. Weibull**, “Balanced-budget Redistribution as the Outcome of Political Competition,” *Public Choice*, 1987, 52, 273–297.
- Lockwood, B.B. and M. Weinzierl**, “Positive and normative judgments implicit in U.S. tax policy, and the costs of unequal growth and recessions,” *Journal of Monetary Economics*, 2016, 14-119 (77), 30–47.
- Meltzer, A. and S. Richard**, “A Rational Theory of the Size of Government,” *Journal of Political Economy*, 1981, 89, 914–927.
- Mirrlees, J.**, “An Exploration in the Theory of Optimum Income Taxation,” *Review of Economic Studies*, 1971, 38, 175–208.
- Myerson, R.**, “Incentives to Cultivate Favored Minorities Under Alternative Electoral Systems,” *American Political Science Review*, 1993, 87 (4), 856–869.
- Peichl, A., N. Pestel, S. Siegloch, and E. Sommer**, “Bundestagswahlkampf 2013: Klientelpolitik durch Steuerreform?..,” *Perspektiven der Wirtschaftspolitik*, 2014, 15 (2), 182–194.
- Persson, T and G. Tabellini**, *Political Economics: Explaining Economic Policy*, Cambridge, MA, MIT Press, 2000.
- Piketty, T.**, “Social mobility and redistributive politics,” *Quarterly Journal of Economics*, 1995, 110 (3), 551–584.
- , “La redistribution fiscale face au chômage,” *Revue française d’économie*, 1997, 12, 157–201.
- and **E. Saez**, “Optimal Labor Income Taxation,” *Handbook of Public Economics*, 2013, pp. 391–474.
- Röell, A.A.**, “Voting over Nonlinear Income Tax Schedules,” *Working Paper*, 2012.
- Roberts, K.**, “Voting over Income Tax Schedules,” *Journal of Public Economics*, 1977, 8.
- Rothstein, P.**, “Order restricted preferences and majority rule,” *Social Choice and Welfare*, 1990, 7 (4), 331–342.
- , “Representative Voter Theorems,” *Public Choice*, 1991, 72 (2/3), 193–212.
- Saez, E.**, “Using Elasticities to Derive Optimal Income Tax Rates,” *Review of Economic Studies*, 2001, 68, 205–229.
- , “Optimal Income Transfer Programs: Intensive versus Extensive Labor Supply Responses,” *Quarterly Journal of Economics*, 2002, 117 (3), 1039–1073.

- , “Do Taxpayers Bunch at Kink Points?,” *American Economic Journal: Economic Policy*, August 2010, 2 (3), 180–212.
- and **S. Stantcheva**, “Generalized Social Marginal Welfare Weights for Optimal Tax Theory,” *American Economic Review*, 2016, 106 (1), 24–45.
- Scheuer, F. and A. Wolitzky**, “Capital taxation under political constraints,” *American Economic Review*, 2016, 106 (8), 2304–2328.
- and **I. Werning**, “Mirrlees meets Diamond-Mirrlees,” Working Paper 22076, National Bureau of Economic Research 2016.
- Scheve, K. and D. Stasavage**, *Taxing the Rich: A History of Fiscal Fairness in the United States and Europe*, Princeton University Press, 2016.
- Seade, J.**, “On the Sign of the Optimum Marginal Income Tax,” *Review of Economic Studies*, 1982, 49 (4), 637–643.
- Sheshinski, E.**, “The Optimal Linear Income-tax,” *Review of Economic Studies*, 1972, 39, 297–302.
- Weinzierl, M.**, “Revisiting the classical view of benefit-based taxation,” *Economic Journal*, forthcoming.
- Weymark, John A.**, “Undominated directions of tax reform,” *Journal of Public Economics*, 1981, 16 (3), 343 – 369.
- Zoutman, F.T., B. Jacobs, and E.L.W. Jongen**, “Optimal Redistributive Taxes and Redistributive Preferences in the Netherlands,” *mimeo: Erasmus University Rotterdam/CPB Netherlands Bureau for Economic Policy Research*, 2014.
- , — , and — , “Redistributive Politics and the Tyranny of the Middle Class,” *Tinbergen Institute Discussion Paper 16-032/VI*, 2016.

Appendix

A Reforms in the (τ, y_a, y_b) -class: Behavioral responses

Reforms in the (τ, y_a, y_b) -class involve jumps of marginal tax rates at y_a and y_b . Lemmas A.1-A.3 below clarify the behavioral responses to these discontinuities. Figure 9 provides an illustration.

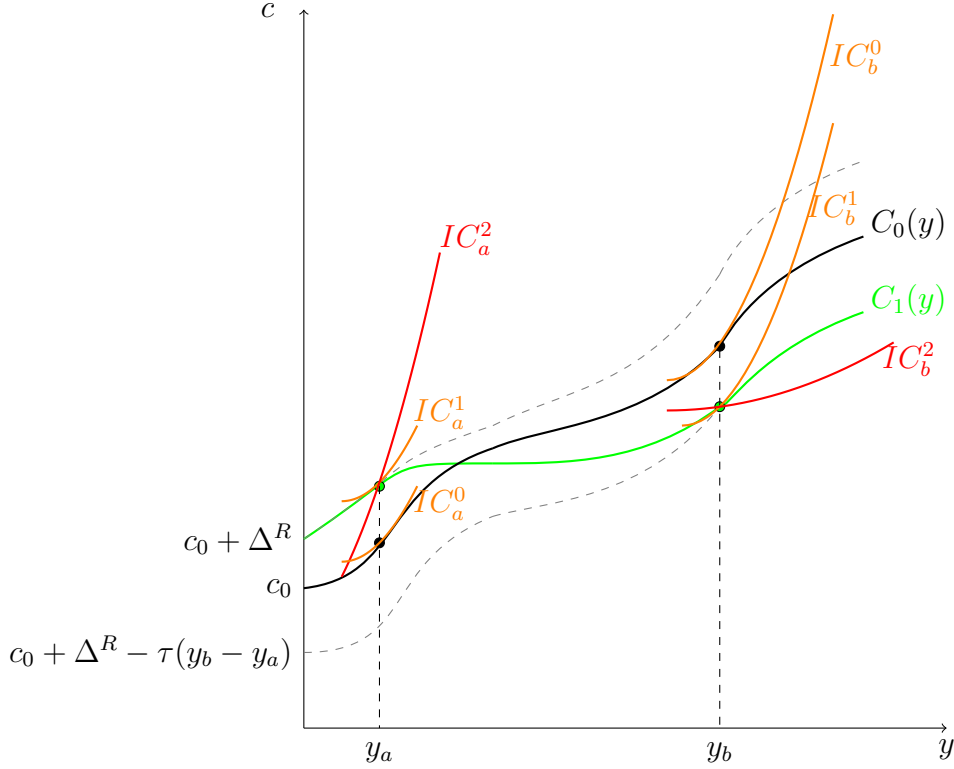


Figure 9:

The figure depicts the following curves:

- (a) $C_0(y) = c_0 + y - T_0(y)$, $C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau h(y)$,
- (b) IC_b^0 indifference curve of type $\omega_b(0)$ through y_b under $C_0(y)$, IC_b^1 curve through y_b under $C_1(y)$ when there are no income effects, IC_b^2 curve through y_b under $C_1(y)$ with income effects;
- (c) IC_a^0 indifference curve of type $\omega_a(0)$ through y_a under $C_0(y)$, IC_a^1 curve through y_a under $C_1(y)$ when there are no income effects, IC_a^2 curve through y_a under $C_1(y)$ if there are income effects.

The Spence-Mirrlees single crossing property implies that, under any tax schedule, more productive individuals choose a higher level of pre-tax income. Thus, $\omega' > \omega$ implies that

$$y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega') \geq y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) ,$$

for $\tau' \in \{0, \tau\}$. We can therefore define threshold types $\omega_a(\tau')$ and $\omega_b(\tau')$ so that

$$\omega \leq \omega_a(\tau') \quad \text{implies} \quad y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) \leq y_a ,$$

$$\omega \in (\omega_a(\tau'), \omega_b(\tau')) \quad \text{implies} \quad y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) \in (y_a, y_b) ,$$

and

$$\omega \geq \omega_b(\tau') \quad \text{implies} \quad y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) \geq y_b .$$

The following Lemma asserts that, for a reform that involves an increase of the marginal tax rate, $\tau > 0$, type $\omega_a(0)$ who chooses an income of y_a before the reform does not choose a level of income above y_a after the reform. Analogously, if marginal taxes go down, type $\omega_b(0)$ does not choose an income above y_b after the reform. For a reform with $\tau > 0$, the logic is as follows: After the reform, because of the transfer Δ^R , a type $\omega_a(0)$ -individual is, at income level y_a , less eager to work more. Working more also is less attractive after the reform because of the increased marginal tax rates for incomes above y_a . Thus, both the income and the substitution effect associated with the reform make it less attractive for a type $\omega_a(0)$ -individual to increase her income above y_a . The individual will therefore either stay at y_a or decrease her income after the reform. Only higher types will end up with an income of y_a after the reform, which implies $\omega_a(0) \leq \omega_a(\tau)$.

Lemma A.1

1. Consider a reform so that $\tau > 0$ and $\Delta^R(\tau, y_a, y_b) > 0$. Then $\omega_a(\tau) \geq \omega_a(0)$.
2. Consider a reform so that $\tau < 0$ and $\Delta^R(\tau, y_a, y_b) < 0$. Then $\omega_b(\tau) \geq \omega_b(0)$.

Proof We only prove the first statement. The proof of the second statement follows from an analogous argument. Under the initial tax schedule T_0 an individual with type $\omega_a(0)$ prefers y_a over all income levels $y \geq y_a$. We argue that the same is true under the new tax schedule T_1 . This proves that $\omega_a(\tau) \geq \omega_a(0)$. Consider a y - c -diagram and the indifference curve of a type $\omega_a(0)$ -type through the point $(y_a, c_0 + y_a - T_0(y_a))$. By definition of type $\omega_a(0)$, all points $(y, y - T_0(y))$ with $y > y_a$ lie below this indifference curve under the initial tax schedule. Under the new schedule T_1 , this individual receives a lump-sum transfer $\Delta^R > 0$. Hence, the indifference curve through $(y_a, c_0 + \Delta^R + y_a - T_0(y_a))$ is at least as steep as the indifference curve through $(y_a, c_0 + y_a - T_0(y_a))$. Thus, the individual prefers $(y_a, c_0 + \Delta^R + y_a - T_0(y_a))$ over all points $(y, c_0 + \Delta^R + y - T_0(y))$ with $y > y_a$. Since for all $y > y_a$, $T_1(y) > T_0(y)$, the individual also prefers $(y_a, c_0 + \Delta^R + y_a - T_0(y_a))$ over all points $(y, c_0 + \Delta^R + y - T_1(y))$ with $y > y_a$. \square

According to the next lemma a reform induces bunching of individuals who face an upward jump of marginal tax rates after the reform. Specifically, a reform with $\tau > 0$ will induce bunching at y_a because marginal tax rates jump upwards at income level y_a . A reform with $\tau < 0$ will induce bunching at y_b because marginal tax rates jump upwards at income level y_b .

Lemma A.2

1. Consider a reform so that $\tau > 0$ and $\Delta^R(\tau, y_a, y_b) > 0$. Then there is a set of types $[\omega_a(\tau), \bar{\omega}_a(\tau)]$ who bunch at y_a after the reform.
2. Consider a reform so that $\tau < 0$ and $\Delta^R(\tau, y_a, y_b) < 0$. Then there is a set of types $[\omega_b(\tau), \bar{\omega}_b(\tau)]$ who bunch at y_b after the reform.

Proof Consider a reform that involves an increase of marginal tax rates $\tau > 0$. If before the reform, all types were choosing an income level $y^*(0, 0, \omega)$ that satisfies the first order condition

$$u_c(\cdot)(1 - T'_0(\cdot)) + u_y(\cdot) = 0 ,$$

then, after the reform there will be a set of types $(\omega_a(\tau), \bar{\omega}(\tau))$ who will now bunch at y_a . These individuals chose an income level above y_a before the reform. After the reform they will find that

$$u_c(\cdot)(1 - T'_0(\cdot) - \tau) + u_y(\cdot) < 0 ,$$

for all $y \in (y_a, y^*(0, 0, \omega)]$ and therefore prefer y_a over any income in this range. At the same time, they will find that

$$u_c(\cdot)(1 - T'_0(\cdot)) + u_y(\cdot) > 0 ,$$

so that there is also no incentive to choose an income level lower than y_a . Hence, the types who bunch are those for which at y_a ,

$$1 - T'_0(y_a) > -\frac{u_y(\cdot)}{u_c(\cdot)} > 1 - T'_0(y_a) - \tau .$$

An analogous argument implies that for a reform that involves a decrease in marginal tax rates $\tau > 0$, there will be a set of types $(\omega_b(\tau), \bar{\omega}_b(\tau))$ who bunch at y_b after the reform. \square

Individuals who bunch at y_a after a reform with $\tau > 0$ neither have an incentive to increase their earnings above y_a since

$$u_c(\cdot)(1 - T'_1(y_a)) + u_y(\cdot) = u_c(\cdot)(1 - T'_0(y_a) - \tau) + u_y(\cdot) \leq 0$$

nor an incentive to lower their earnings since

$$u_c(\cdot)(1 - T'_0(y_a)) + u_y(\cdot) \geq 0 .$$

By contrast, there are no individuals who bunch at y_b after a reform that involves increased marginal tax rates. Individuals who do not have an incentive to increase their income at y_b since

$$u_c(\cdot)(1 - T'_1(y_b)) + u_y(\cdot) = u_c(\cdot)(1 - T'_0(y_b)) + u_y(\cdot) \leq 0$$

definitely have an incentive to lower their income as

$$u_c(\cdot)(1 - T'_0(y_b) - \tau) + u_y(\cdot) < 0 .$$

An analogous argument implies that no one will bunch at y_a after a reform that involves a decrease of marginal tax rates.

The next Lemma establishes that, in the absence of income effects, for a reform with $\tau > 0$, individuals who choose an income above y_b after the reform also chose an income above y_b before the reform. This ordering is reversed for a reform with $\tau < 0$. We will subsequently discuss why these statements need no longer be true if there are income effects.

Lemma A.3 *Suppose that there are no income effects, i.e. for all (c, y, ω) and any pair (e, e') with $e' > e$,*

$$-\frac{u_y(c + e, y, \omega')}{u_c(c + e, y, \omega')} = -\frac{u_y(c + e', y, \omega)}{u_c(c + e', y, \omega)}.$$

1. *Consider a reform so that $\tau > 0$ and $\Delta^R(\tau, y_a, y_b) > 0$. Then $\omega_b(\tau) \geq \omega_b(0)$.*
2. *Consider a reform so that $\tau < 0$ and $\Delta^R(\tau, y_a, y_b) < 0$. Then $\omega_a(\tau) \geq \omega_a(0)$.*

Proof We only prove the first statement. The proof of the second statement follows from an analogous argument. Under the initial tax schedule T_0 an individual with type $\omega_b(0)$ prefers y_b over all income levels $y \geq y_b$. We argue that the same is true under the new tax schedule T_1 . This proves that $\omega_b(\tau) \geq \omega_b(0)$. Consider a y - c -diagram and the indifference curve of a type $\omega_b(0)$ -type through the point $(y_b, c_0 + y_b - T_0(y_b))$. By definition of type $\omega_b(0)$, all points $(y, c_0 + y - T_0(y))$ with $y > y_b$ lie below this indifference curve under the initial tax schedule. Under the new schedule T_1 , this individual receives a lump-sum transfer $\Delta^R - \tau(y_b - y_a) < 0$. Without income effects, the indifference curve through $(y_b, c_0 + \Delta^R - \tau(y_b - y_a) + y_b - T_0(y_b))$ has the same slope as the indifference curve through $(y_b, c_0 + y_b - T_0(y_b))$. Thus, the individual prefers $(y_b, c_0 + \Delta^R - \tau(y_b - y_a) + y_b - T_0(y_b))$ over all points $(y, c_0 + \Delta^R - \tau(y_b - y_a) + y - T_0(y))$ with $y > y_b$, or equivalently over all points $(y, c_0 + \Delta^R + y - T_1(y))$ with $y > y_b$. \square

If there are no income effects, then, after a reform with $\tau > 0$, an individual with type $\omega_b(0)$ prefers an income level of y_b over any income above y_b before and after the reform since (i) the indifference curve through $(c_0 + y_b - T_0(y_b), y_b)$ has the same slope as the indifference curve through $(c_0 + \Delta^R + y_b - T_1(y_b), y_b)$ and (ii) for $y > y_b$, $T_0'(y) = T_1'(y)$ so that the incentives to increase income above y_b are unaffected by the reform. The individual has, however, an incentive to lower y since, because of the increased marginal tax rate, working less has become cheaper; i.e. it is no longer associated with as big a reduction of consumption. Thus, $\omega_b(0) \leq \omega_b(\tau)$ if there are no income effects. Figure 9 provides an illustration. With income effects there is also an opposing force since the individual also has to pay additional taxes $\tau(y_b - y_a) - \Delta^R$ which tends to flatten the indifference curve through y_b . Thus, there may both be income levels below y_b and income levels above y_b that the individual prefers over y_b . If the indifference curve flattens a lot, the individual will end up choosing $y > y_b$ after the reform which implies that $\omega_b(\tau) < \omega_b(0)$.

B Proofs

Proof of Theorem 1

Step 1. We show that

$$\Delta_\tau^V(\omega | 0, h) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega))) ,$$

for all $\omega \in \Omega$, where $\tilde{u}_c^0(\omega)$ is a shorthand for the marginal utility of consumption that a type ω -individual realizes in the status quo.

For individuals whose behavior is characterized by a first-order condition, this follows from a straightforward application of the envelope theorem. If we consider reforms in the (τ, y_a, y_b) -class we have to take account of the possibility of bunching. Consider the case with $\tau > 0$. The case $\tau < 0$ is analogous. With $\tau > 0$, there will be individuals who bunch at y_a . For these individuals, marginal changes of Δ^R and τ do not trigger an adjustment of the chosen level of earnings. Hence,

$$\begin{aligned}\Delta_\tau^V(\omega \mid \tau, h) &= \frac{d}{d\tau} u(c_0 + \Delta^R(\tau, h) + y_a - T_0(y_a) - \tau h(y_a), y_a, \omega) \\ &= u_c(c_0 + \Delta^R(\tau, h) + y_a - T_0(y_a) - \tau h(y_a), y_a, \omega) (\Delta_\tau^R(\tau, h) - h(y_a))\end{aligned}$$

By assumption, the status quo tax schedule does not induce bunching. Thus, at $\tau = 0$, this expression applies only to type $\omega_a(0)$ for whom $\tilde{y}^0(\omega_a(0)) = y_a$. This proves that

$$\Delta_\tau^V(\omega \mid 0, h) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega))) ,$$

holds for all ω .

Step 2. Suppose that h is a non-decreasing function. An analogous argument applies if h is non-increasing. We show that $\Delta_\tau^V(\omega^M \mid 0, h) > 0$ implies $\Delta_\tau^V(\omega \mid 0, h) > 0$ for a majority of individuals.

By Step 1, $\Delta_\tau^V(\omega^M \mid 0, h) > 0$ holds if and only if $\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega^M)) > 0$. As h and \tilde{y} are non-decreasing functions, this implies $\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega)) > 0$, for all $\omega \leq \omega^M$, and hence $\Delta_\tau^V(\omega \mid 0, h) > 0$ for all $\omega \leq \omega^M$.

Step 3. Suppose that h is a non-decreasing function. An analogous argument applies if h is non-increasing. We show that $\Delta_\tau^V(\omega^M \mid 0, h) \leq 0$ implies $\Delta_\tau^V(\omega \mid 0, h) \leq 0$ for a majority of individuals.

By Step 1, $\Delta_\tau^V(\omega^M \mid 0, h) \leq 0$ holds if and only if $\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega^M)) \leq 0$. As h and \tilde{y} are non-decreasing functions, this implies $\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega)) \leq 0$, for all $\omega \geq \omega^M$, and hence $\Delta_\tau^V(\omega \mid 0, h) \leq 0$ for all $\omega \geq \omega^M$.

Proof of Lemma 1

For individuals whose behavior is characterized by a first-order condition, the utility realized under a reform that involves a change in the lump-sum-transfer by Δ^R and a change of marginal taxes by τ is given by $V(\Delta^R, \tau, \omega)$. The marginal rate of substitution between τ and Δ^R is therefore given by

$$\left(\frac{d\Delta^R}{d\tau} \right)_{|dV=0} = - \frac{V_\tau(\Delta^R, \tau, \omega)}{V_e(\Delta^R, \tau, \omega)}.$$

By the envelope theorem,

$$V_\tau(\Delta^R, \tau, \omega) = -u_c(\cdot) h(y^*(\Delta^R, \tau, \omega)) \quad \text{and} \quad V_e(\Delta^R, \tau, \omega) = u_c(\cdot) .$$

Thus,

$$\left(\frac{d\Delta^R}{d\tau} \right)_{|dV=0} = h(y^*(\Delta^R, \tau, \omega)) .$$

If we consider reforms in the (τ, y_a, y_b) -class we have to take account of the possibility of bunching. We only consider the case with $\tau > 0$. The case $\tau < 0$ is analogous. With $\tau > 0$, there will be individuals who bunch at y_a . For these individuals, marginal changes of Δ^R and τ do not trigger an adjustment of the chosen level of earnings. Hence, a marginal change of Δ^R yields a change in utility equal to $u_c(\cdot)$. The change in utility due to a change in the marginal tax rate is given by $-u_c(\cdot) h(y_a) = 0$. Again, the marginal rate of substitution equals $h(y_a) = 0$.

Proof of Proposition 1

From Step 1 in the proof of Theorem 1 we know that

$$\Delta_\tau^V(\omega \mid 0, h) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega))) .$$

Now let

$$\Delta_\tau^V(\omega^M \mid 0, h) = \tilde{u}_c^0(\omega^M) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega^M))) > 0 .$$

With h non-increasing for $y \geq y^M$, this implies that

$$\Delta_\tau^V(\omega \mid 0, h) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega))) > 0 ,$$

for all $\omega \geq \omega^M$ and hence for a majority of the population.

Proof of Proposition 2

We omit the proof of Proposition 2 as the formal argument is identical to the one for Proposition 1.

Proof of Lemma 2

We seek to show that, for all ω ,

$$\Delta_\tau^V(\omega \mid \tau, h) = \tilde{u}_c^1(\omega) (\Delta_\tau^R(\tau, h) - h(\tilde{y}^1(\omega))) .$$

Suppose that $\tau > 0$. The case $\tau < 0$ is analogous. We first consider reforms (τ, h) under which individual earnings are characterized by a first order condition. In this case, the envelope theorem implies

$$\Delta_\tau^V(\omega \mid \tau, h) = u_c(\cdot) (\Delta_\tau^R(\tau, h) - h(\tilde{y}^1(\omega))) ,$$

where $u_c(\cdot)$ is marginal utility evaluated at $c = c_0 + \Delta^R(\tau, h) + \tilde{y}^1(\omega) - T_1(\tilde{y}^1(\omega))$ and $\tilde{y}^1(\omega) = y^*(\Delta^R(\tau, h), \tau, \omega)$.

We now consider a reform that belongs to the (τ, y_a, y_b) -class. The above reasoning extends to types with $\omega \leq \omega_a(\tau)$, $\omega \in [\bar{\omega}_a(\tau), \omega_b(\tau)]$ and $\omega \geq \omega_b(\tau)$ whose behavior is characterized by a first order condition. Individuals with types in $[\omega_a(\tau), \bar{\omega}_a(\tau)]$ bunch at income level y_a after the reform so that

$$\Delta^V(\omega \mid \tau, y_a, y_b) = u(c_0 + \Delta^R(\tau, y_a, y_b) + y_a - T_0(y_a), y_a, \omega) - V(0, 0, \omega) .$$

Hence,

$$\Delta_{\tau}^V(\omega \mid \tau, y_a, y_b) = u_c(\cdot) \Delta_{\tau}^R(\tau, y_a, y_b) ,$$

where $u_c(\cdot)$ is marginal utility evaluated at $c = c_0 + \Delta^R(\tau, y_a, y_b) + y - T_0(y)$ and $y = y_a$. Since $h(y_a) = 0$, this implies, in particular,

$$\Delta_{\tau}^V(\omega \mid \tau, y_a, y_b) = u_c(\cdot) (\Delta_{\tau}^R(\tau, y_a, y_b) - h(y_a)) .$$

Proof of Proposition 3

Suppose that h is a non-decreasing function. The proof for h non-increasing is analogous. By (4),

$$\Delta_{\tau}^V(\omega^M \mid \tau', h) > 0$$

for all $\tau' \in (0, \tau)$, implies

$$\Delta_{\tau}^R(\tau', h) - h(\tilde{y}^1(\omega^M)) > 0,$$

for all $\tau' \in (0, \tau)$. By the Spence-Mirrlees single crossing property \tilde{y}^1 is a non-decreasing function. Hence, $\omega \leq \omega^M$ implies

$$\Delta_{\tau}^R(\tau', h) - h(\tilde{y}^1(\omega)) > 0,$$

for all $\tau' \in (0, \tau)$ and, therefore, by (3) and (4)

$$\Delta^V(\omega \mid \tau', h) > 0 .$$

Thus, the reform is supported by a majority of the population. This proves the first statement in Proposition 3. The second statement follows from an analogous argument.

Proof of Lemma 3

We show that, for all ω ,

$$\tilde{u}_c^{1'}(\omega) := \frac{\partial}{\partial \omega} \tilde{u}_c^1(\omega) \leq 0 .$$

We also define

$$\tilde{y}^{1'}(\omega) := \frac{\partial}{\partial \omega} \tilde{y}^1(\omega) .$$

By the Spence-Mirrlees single crossing property, \tilde{y}^1 is a non-decreasing function: $\tilde{y}^{1'}(\omega) \geq 0$ for all ω . By definition,

$$\tilde{u}_c^1(\omega) = u_c(c_0 + \Delta^R + \tilde{y}^1(\omega) - T_1(\tilde{y}^1(\omega)), \tilde{y}^1(\omega), \omega) ,$$

where $\tilde{y}^1(\omega)$ equals y_a for a reform in the (τ, y_a, y_b) -class and $\omega \in [\omega_a(\tau), \bar{\omega}_a(\tau)]$ and equals $y^*(\Delta^R, \tau, \omega)$ otherwise. Hence,

$$\tilde{u}_c^{1'}(\omega) = (u_{cc}(\cdot)(1 - T_1'(\cdot)) + u_{cy}(\cdot)) \tilde{y}^{1'}(\omega) + u_{c\omega}(\cdot) .$$

If the individual is bunching at y_a , then $\tilde{y}^1(\omega) = 0$, which implies

$$\tilde{u}_c^1(\omega) = u_{c\omega}(\cdot) \leq 0 .$$

It remains to be shown that $\tilde{u}_c^1(\omega) \leq 0$ also holds if the individual is not bunching at y_a . If the individual is not bunching at y_a , $\tilde{y}^1(\omega)$ satisfies the first order condition

$$1 - T_1'(\cdot) = -\frac{u_y(\cdot)}{u_c(\cdot)} .$$

Hence,

$$\tilde{u}_c^1(\omega) = \left(-u_{cc}(\cdot) \frac{u_y(\cdot)}{u_c(\cdot)} + u_{cy}(\cdot) \right) \tilde{y}^1(\omega) + u_{c\omega}(\cdot) ,$$

where, because of (1),

$$-u_{cc}(\cdot) \frac{u_y(\cdot)}{u_c(\cdot)} + u_{cy}(\cdot) \leq 0 .$$

Proof of Theorem 2

For a small reform in the (τ, y_a, y_b) -class, evaluated at the status quo, equation (4) implies that

$$\Delta_\tau^V(\omega \mid 0, y_a, y_b) = \tilde{u}_c^0(\omega) (\Delta_\tau^R(0, y_a, y_b) - h(\tilde{y}^0(\omega))) , \quad (19)$$

To prove the first statement in Theorem 2, suppose that $y_0 < \tilde{y}^0(w^M)$. Choose y_a and y_b so that $y_a < y_0 < y_b < \tilde{y}^0(w^M)$. For a reform in the (τ, y_a, y_b) -class, $y_b < \tilde{y}^0(w^M)$ implies $h(\tilde{y}^0(\omega)) = y_b - y_a$, for all $\omega \geq \omega^M$. Since $\Delta_\tau^R(0, y_a, y_b) < y_b - y_a$, it follows that $\Delta_\tau^V(\omega \mid 0, y_a, y_b) < 0$, for all $\omega \geq \omega^M$, which implies that a small tax cut makes a majority of individuals better off.

To prove the second statement, suppose that $y_0 > \tilde{y}^0(w^M)$ and choose y_a and y_b so that $\tilde{y}^0(w^M) < y_a < y_0 < y_b$. For a reform in the (τ, y_a, y_b) -class, $\tilde{y}^0(w^M) < y_a$ implies $h(\tilde{y}^0(\omega)) = 0$, for all $\omega \leq \omega^M$. Hence, if $\Delta_\tau^R(0, y_a, y_b) > 0$, then $\Delta_\tau^V(\omega \mid 0, y_a, y_b) > 0$, for all $\omega \leq \omega^M$, which implies that a small tax raise makes a majority of individuals better off.

Proof of Proposition 4

We provide a proof of the first statement in the Proposition by considering a reform with $\tau > 0$. The second statement follows from an analogous argument.

Notation. We look at different subsets of the population separately. The change in tax revenue that is due to individuals with types $\omega \leq \omega_a(\tau)$ who chose an income level smaller or equal to y_a after the reform is given by

$$\begin{aligned} \Delta^{R1}(\tau, y_a, y_b) &:= \int_{\underline{\omega}}^{\omega_a(\tau)} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\underline{\omega}}^{\omega_a(\tau)} \{T_0(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\underline{\omega}}^{\omega_a(\tau)} \{T_0(y^*(\Delta^R(\cdot), 0, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega , \end{aligned}$$

where the last equality uses the fact that the behavior of individuals with types in $\omega_a(\tau)$ is affected only by $\Delta^R(\cdot)$ but not by the increased marginal tax rates for individuals with incomes in $[y_a, y_b]$.

The change in tax revenue that comes from individuals with types in $[\omega_a(\tau), \bar{\omega}_a(\tau)]$ who bunch at an income level of y_a after the reform equals

$$\begin{aligned}\Delta^{R2}(\tau, y_a, y_b) &:= \int_{\omega_a(\tau)}^{\bar{\omega}_a(\tau)} \{T_1(y_a) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\omega_a(\tau)}^{\bar{\omega}_a(\tau)} \{T_0(y_a) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega.\end{aligned}$$

Individuals with types in $(\bar{\omega}_a(\tau), \omega_b(\tau))$ choose an income level between y_a and y_b after the reform. The change in tax revenue that can be attributed to them equals

$$\begin{aligned}\Delta^{R3}(\tau, y_a, y_b) &:= \int_{\bar{\omega}_a(\tau)}^{\omega_b(\tau)} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\bar{\omega}_a(\tau)}^{\omega_b(\tau)} \{T_0(y^*(\Delta^R(\cdot), \tau, \omega)) + \tau(y^*(\Delta^R(\cdot), \tau, \omega) - y_a) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega.\end{aligned}$$

Finally, the change in tax revenue that comes from individuals with types above $\omega_b(\tau)$ who choose an income larger than y_b after the reform is given by

$$\begin{aligned}\Delta^{R4}(\tau, y_a, y_b) &:= \int_{\omega_b(\tau)}^{\bar{\omega}} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\omega_b(\tau)}^{\bar{\omega}} \{T_0(y^*(\Delta^R(\cdot), \tau, \omega)) + \tau(y_b - y_a) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega \\ &= \int_{\omega_b(\tau)}^{\bar{\omega}} \{T_0(y^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega)) + \tau(y_b - y_a) - T_0(y^*(0, 0, \omega))\} f(\omega) d\omega,\end{aligned}$$

where the last equality uses the fact that the behavior of individuals with types above $\omega_b(\tau)$ is affected only by the transfer $\Delta^R(\cdot) - \tau(y_b - y_a)$ but not by the increased marginal tax rates for individuals with incomes in $[y_a, y_b]$, see Figure 1. To sum up,

$$\Delta^R(\tau, y_a, y_b) = \Delta^{R1}(\tau, y_a, y_b) + \Delta^{R2}(\tau, y_a, y_b) + \Delta^{R3}(\tau, y_a, y_b) + \Delta^{R4}(\tau, y_a, y_b).$$

How does a marginal change of τ affect tax revenue? In the following we will provide a characterization of $\Delta_\tau^R(\tau, y_a, y_b)$, i.e. of the change in tax revenue that is implied by a marginal change of the tax rate τ . We will be particularly interested in evaluating this expression at $\tau = 0$ since $\Delta_\tau^R(0, y_a, y_b)$ gives the increase in tax revenue from a small increase of marginal tax rates for incomes in the interval $[y_a, y_b]$. Since

$$\Delta_\tau^R(\tau, y_a, y_b) = \Delta_\tau^{R1}(\tau, y_a, y_b) + \Delta_\tau^{R2}(\tau, y_a, y_b) + \Delta_\tau^{R3}(\tau, y_a, y_b) + \Delta_\tau^{R4}(\tau, y_a, y_b),$$

we can characterize $\Delta_\tau^R(\tau, y_a, y_b)$ by looking at each subset of types separately.

For instance, it is straightforward to verify that

$$\begin{aligned}\Delta_\tau^{R1}(\tau, y_a, y_b) &= \Delta_\tau^R(\cdot) \int_{\omega_a(\tau)}^{\omega_a(\tau)} T_0'(y^*(\Delta^R(\cdot), 0, \omega)) y_e^*(\Delta^R(\cdot), 0, \omega) f(\omega) d\omega \\ &\quad + \{T_0(y_a) - T_0(y^*(0, 0, \omega_a(\tau)))\} f(\omega_a(\tau)) \omega_a'(\tau)\end{aligned}$$

where $\omega_a'(\tau)$ is the derivative of $\omega_a(\tau)$. Analogously we derive expressions for $\Delta_\tau^{R2}(\tau, y_a, y_b)$, $\Delta_\tau^{R3}(\tau, y_a, y_b)$, $\Delta_\tau^{R4}(\tau, y_a, y_b)$. This is tedious, but straightforward. We provide the details in Appendix D. It yields an expression for $\Delta_\tau^R(\tau, y_a, y_b)$. If we evaluate this expression at $\tau = 0$ and use Assumption 1, i.e. the assumption that there is no bunching at y_a under the initial tax schedule T_0 , so that $\omega_a(0) = \hat{\omega}_a(0)$, we obtain

$$\begin{aligned}\Delta_\tau^R(0, y_a, y_b) &= \Delta_\tau^R(0, y_a, y_b) I_0 \\ &\quad + \int_{\omega_a(0)}^{\omega_b(0)} T_0'(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega \\ &\quad + \int_{\omega_a(0)}^{\omega_b(0)} \{y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad + (y_b - y_a)(1 - F(\omega_b(0))) \\ &\quad - (y_b - y_a) \int_{\omega_b(0)}^{\bar{\omega}} T_0'(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega,\end{aligned}$$

where $I_0 := \tilde{I}(\underline{\omega})$ is the income effect measure \tilde{I} defined in the body of the text, see equation (6), applied to the population at large.

Armed with this notation, we can write

$$\Delta_\tau^R(0, y_a, y_b) = \frac{1}{1 - I_0} \mathcal{R}(y_a, y_b),$$

where

$$\begin{aligned} \mathcal{R}(y_a, y_b) &= \int_{\omega_a(0)}^{\omega_b(0)} T_0'(y^*(0, 0, \omega)) y_\tau^*(0, 0, \omega) f(\omega) d\omega \\ &\quad + \int_{\omega_a(0)}^{\omega_b(0)} \{y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad + (y_b - y_a)(1 - F(w_b)) \\ &\quad - (y_b - y_a) \int_{\omega_b(0)}^{\bar{\omega}} T_0'(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega. \end{aligned}$$

In the following, to simplify notation, we suppress the dependence of $\omega_a(0)$, $\omega_b(0)$ on $\tau = 0$ and simply write ω_a , and ω_b . By assumption, there is no bunching under the initial schedule, so that $y^*(0, 0, \omega)$ is strictly increasing and hence invertible over $[\omega_a, \omega_b]$. We can therefore, without loss of generality, view a reform also as being defined by τ , ω_a and ω_b . With a slight abuse of notation, we will therefore write

$$\Delta_\tau^R(0, \omega_a, \omega_b) = \frac{1}{1 - I_0} \mathcal{R}(\omega_a, \omega_b) \tag{20}$$

where

$$\begin{aligned} \mathcal{R}(\omega_a, \omega_b) &= \int_{\omega_a}^{\omega_b} T_0'(y^*(0, 0, \omega)) y_\tau^*(0, 0, \omega) f(\omega) d\omega \\ &\quad + \int_{\omega_a}^{\omega_b} \{y^*(0, 0, \omega) - y^*(0, 0, \omega_a)\} f(\omega) d\omega \\ &\quad + (y^*(0, 0, \omega_b) - y^*(0, 0, \omega_a))(1 - F(w_b(0))) \\ &\quad - (y^*(0, 0, \omega) - y^*(0, 0, \omega_a)) \int_{\omega_b}^{\bar{\omega}} T_0'(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega \end{aligned}$$

We now investigate under which conditions a marginal tax increase of τ over a small interval of types increases tax revenue. Note that

$$\Delta_\tau^R(0, \omega_a, \omega_a) = \frac{1}{1 - I_0} \mathcal{R}(\omega_a, \omega_a) = 0.$$

If

$$\Delta_{\tau\omega_b}^R(0, \omega_a, \omega_a) = \frac{1}{1 - I_0} \mathcal{R}_{\omega_b}(\omega_a, \omega_a) > 0,$$

then $\Delta_\tau^R(0, \omega_a, \omega_b)$ turns positive, if starting from $\omega_a = \omega_b$, we marginally increase ω_b .

Straightforward computations yield:

$$\mathcal{R}_{\omega_b}(\omega_a, \omega_a) = T_0'(y^*(0, 0, \omega_a)) y_\tau^*(0, 0, \omega_a) f(\omega_a) + (1 - \tilde{I}_0(\omega_a)) y_\omega^*(0, 0, \omega_a) (1 - F(\omega_a)).$$

We summarize these observations in the following Lemma:

Lemma B.1 *Suppose that, under tax schedule T_0 , there is an income level y_0 and a type ω_0 with $y^*(0, 0, \omega_0) = y_0$ so that*

$$T_0'(y^*(0, 0, \omega_0)) y_\tau^*(0, 0, \omega_0) f(\omega_0) + (1 - \tilde{I}_0(\omega_0)) y_\omega^*(0, 0, \omega_0) (1 - F(\omega_0)) > 0.$$

Then there exists a revenue-increasing tax reform (τ, y_a, y_b) with $\tau > 0$, $y_a < y_0 < y_b$.

On the sign of $y_\tau^*(0, 0, \omega_0)$. In the following, we argue that in Lemma B.1 above, we have $y_\tau^*(0, 0, \omega_0) < 0$. To see this, consider the optimization problem

$$\max_y u(c_0 + y - T_0(y) - \tau(y - y_a), y, \omega_0)$$

for type ω_0 . By assumption $y^*(0, 0, \omega_0) \in (y_a, y_b)$. We argue that for any τ so that $y^*(0, \tau, \omega_0) \in (y_a, y_b)$ and $y_b - y_a$ sufficiently small, we have $y_\tau^*(0, 0, \omega_0) < 0$. The first order condition of the optimization problem is

$$u_c(\cdot)(1 - T_0'(\cdot) - \tau) + u_y(\cdot) = 0.$$

The second order condition is, assuming a unique optimum,

$$B := u_{cc}(\cdot)(1 - T_0'(\cdot) - \tau)^2 + 2u_{cy}(\cdot)(1 - T_0'(\cdot) - \tau) + u_{yy}(\cdot) - u_c(\cdot)T_0''(\cdot) < 0.$$

From totally differentiating the first order condition with respect to c_0 , we obtain

$$y_e^*(0, \tau, \omega_0) = -\frac{u_{cc}(\cdot)(1 - T_0'(\cdot) - \tau) + u_{cy}(\cdot)}{B} \leq 0.$$

This expression is non-positive by our assumptions on the utility function that ensure that leisure is a non-inferior good. From totally differentiating the first order condition with respect to τ , and upon collecting terms, we obtain

$$y_\tau^*(0, \tau, \omega_0) = \frac{u_c(\cdot)}{B} - (y^*(0, \tau, \omega_0) - y_a)y_e^*(0, \tau, \omega_0),$$

and hence

$$y_\tau^*(0, 0, \omega_0) = \frac{u_c(\cdot)}{B} - (y^*(0, 0, \omega_0) - y_a)y_e^*(0, 0, \omega_0),$$

which is the familiar decomposition of a behavioral response into a substitution and an income effect. As ω_0 approaches ω_a , $y^*(0, 0, \omega_0)$ approaches $y^*(0, 0, \omega_a) = y_a$ so that the income effect vanishes. Again, this is a familiar result: For small price changes, observed behavioral responses are well approximated by compensated or Hicksian behavioral responses.

The observation that for y_a close to y_b , we may, without loss of generality, assume that $y_\tau^*(0, 0, \omega_0) < 0$, enables us to rewrite Lemma B.1.

Lemma B.2 *Suppose that, under tax schedule T_0 , there is an income level y_0 and a type ω_0 with $y^*(0, 0, \omega_0) = y_0$ so that*

$$T_0'(y_0) < -\frac{1 - F(\omega_0)}{f(\omega_0)} \left(1 - \tilde{I}_0(\omega_0)\right) \frac{y_\omega^*(0, 0, \omega_0)}{y_\tau^*(0, 0, \omega_0)}.$$

Then there exists a tax-revenue-increasing reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.

The right-hand-side of this inequality equals $D^R(\omega_0) = D^R(\tilde{\omega}^0(y_0)) = \mathcal{D}^R(y_0)$. This proves the first statement in Proposition 4.

Proof of Proposition 5

Starting from $\tau = 0$ a tax increase is Pareto-improving if $\Delta_\tau^R(0, y_a, y_b) - (y_b - y_a) \geq 0$, where we recall that

$$\Delta_\tau^R(0, y_a, y_b) = \frac{1}{1 - I_0} \mathcal{R}(y_a, y_b),$$

and

$$\begin{aligned} \mathcal{R}(y_a, y_b) &= \int_{\omega_a}^{\omega_b} T'_0(y^*(0, 0, \omega)) y_\tau^*(0, 0, \omega) f(\omega) d\omega \\ &\quad + \int_{\omega_a}^{\omega_b} \{y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad + (y_b - y_a)(1 - F(\omega_b)) \\ &\quad - (y_b - y_a) \int_{\omega_b}^{\bar{\omega}} T'_0(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega. \end{aligned}$$

Again, we exploit the assumption that there is no bunching under the initial schedule, so that $y^*(0, 0, \omega)$ is strictly increasing and hence invertible over $[\omega_a, \omega_b]$. We can therefore, without loss of generality, view a reform also as being defined by τ , ω_a and ω_b . With a slight abuse of notation, we therefore need to check whether

$$\frac{1}{1 - I_0} \mathcal{R}(\omega_a, \omega_b) - (y^*(0, 0, \omega_b) - y^*(0, 0, \omega_a)) \geq 0,$$

where

$$\begin{aligned} \mathcal{R}(\omega_a, \omega_b) &= \int_{\omega_a}^{\omega_b} T'_0(y^*(0, 0, \omega)) y_\tau^*(0, 0, \omega) f(\omega) d\omega \\ &\quad + \int_{\omega_a}^{\omega_b} \{y^*(0, 0, \omega) - y^*(0, 0, \omega_a)\} f(\omega) d\omega \\ &\quad + (y^*(0, 0, \omega_b) - y^*(0, 0, \omega_a))(1 - F(\omega_b)) \\ &\quad - (y^*(0, 0, \omega_b) - y^*(0, 0, \omega_a)) \int_{\omega_b}^{\bar{\omega}} T'_0(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega. \end{aligned}$$

Note that

$$\begin{aligned} &\Delta_\tau^R(0, \omega_a, \omega_a) - (y^*(0, 0, \omega_a) - y^*(0, 0, \omega_a)) \\ &= \frac{1}{1 - I_0} \mathcal{R}(\omega_a, \omega_a) - (y^*(0, 0, \omega_a) - y^*(0, 0, \omega_a)) \\ &= 0. \end{aligned}$$

Thus, if

$$\frac{1}{1 - I_0} \mathcal{R}_{\omega_b}(\omega_a, \omega_a) - y_\omega^*(0, 0, \omega_a) > 0,$$

then $\Delta_\tau^R(0, \omega_a, \omega_b) - (y^*(0, 0, \omega_b) - y^*(0, 0, \omega_a))$ turns positive, if starting from $\omega_a = \omega_b$, we marginally increase ω_b .

Straightforward computations yield:

$$\begin{aligned} &\frac{1}{1 - I_0} \mathcal{R}_{\omega_b}(\omega_a, \omega_a) - y_\omega^*(0, 0, \omega_a) \\ &= \frac{1}{1 - I_0} \left\{ T'_0(y^*(0, 0, \omega_a)) y_\tau^*(0, 0, \omega_a) f(\omega_a) + (1 - \tilde{I}_0(\omega_a)) y_\omega^*(0, 0, \omega_a) (1 - F(\omega_a)) \right\} - y_\omega^*(0, 0, \omega_a). \end{aligned}$$

Hence, if this expression is positive we can Pareto-improve by increasing marginal tax rates in a neighborhood of y_a . Upon noting that $y_\tau^*(0, 0, \omega_a) < 0$, the statement $\frac{1}{1 - I_0} \mathcal{R}_{\omega_b}(\omega_a, \omega_a) - y_\omega^*(0, 0, \omega_a) > 0$ is easily seen to be equivalent to the claim $T'_0(y_a) < \mathcal{D}^P(y_a)$ in Proposition 5.

Proof of Proposition 7

We only prove the first statement in Proposition 7, the second follows from an analogous argument.

Recall that by equation (4)

$$\Delta_\tau^V(\omega \mid \tau, y_a, y_b) = \tilde{u}_c^1(\omega) (\Delta_\tau^R(\tau, y_a, y_b) - h(\tilde{y}^1(\omega))) ,$$

where $\tilde{u}_c^1(\omega)$ is a shorthand for the marginal utility of consumption that a type ω individual realizes after the reform $\tilde{y}^1(\omega)$ is the income level chosen after the reform. Note that $\tilde{y}^1(\omega)$ equals y_a for types who bunch at y_a , and $y^*(\Delta^R(\tau, y_a, y_b), \tau, \omega)$ otherwise. Consequently,

$$\begin{aligned} \Delta_\tau^W(\tau, y_a, y_b \mid g) &:= \int_{\underline{\omega}}^{\bar{\omega}} g(\omega) \tilde{u}_c^1(\omega) (\Delta_\tau^R(\tau, y_a, y_b) - h(\tilde{y}^1(\omega))) f(\omega) d\omega . \\ &= \gamma(\tau, y_a, y_b) \Delta_\tau^R(\tau, y_a, y_b) \\ &\quad - \int_{\omega_a(\tau)}^{\omega_b(\tau)} g(\omega) \tilde{u}_c^1(\cdot) \{y^*(\Delta^R(\cdot), \tau, \omega) - y_a\} f(\omega) d\omega \\ &\quad - (1 - F(\omega_b(\tau)))(y_b - y_a) \Gamma(\omega_b(\tau) \mid \tau, y_a, y_b) , \end{aligned}$$

where

$$\gamma(\tau, y_a, y_b) := \int_{\underline{\omega}}^{\bar{\omega}} g(\omega) \tilde{u}_c^1(\omega) f(\omega) d\omega ,$$

and

$$\Gamma(\omega_b(\tau) \mid \tau, y_a, y_b) := \int_{\omega_b(\tau)}^{\bar{\omega}} g(\omega) \tilde{u}_c^1(\omega) \frac{f(\omega)}{1 - F(\omega_b(\tau))} d\omega .$$

If we evaluate these welfare weights for $\tau = 0$, we obtain the welfare weights under the initial allocation. It will be convenient to have a more concise notation available. In the following, we will simply write γ_0 for the average welfare weight under the initial allocation, and $\Gamma_0(\omega_b)$ for the average welfare weight among individuals with types larger or equal to $\omega_b(0)$. Thus,

$$\begin{aligned} \Delta_\tau^W(0, y_a, y_b \mid g) &= \gamma_0 \Delta_\tau^R(0, y_a, y_b) \\ &\quad - \int_{\omega_a(0)}^{\omega_b(0)} g(\omega) \tilde{u}_c^0(\cdot) \{y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad - (1 - F(\omega_b(0)))(y_b - y_a) \Gamma_0(\omega_b) , \end{aligned}$$

In the following, to simplify notation, we suppress the dependence of $\omega_a(0)$, $\omega_b(0)$ on $\tau = 0$ and simply write ω_a and ω_b . By assumption, there is no bunching under the initial schedule, so that $y^*(0, 0, \omega)$ is strictly increasing and hence invertible over $[\omega_a, \omega_b]$. We can therefore, without loss of generality, view a reform also as being defined by τ , ω_a and ω_b . With a slight abuse of notation, we will therefore write

$$\begin{aligned} \Delta_\tau^W(0, \omega_a, \omega_b \mid g) &= \gamma_0 \Delta_\tau^R(0, \omega_a, \omega_b) \\ &\quad - \int_{\omega_a}^{\omega_b} g(\omega) \tilde{u}_c^0(\cdot) \{y^*(0, 0, \omega) - y^*(0, 0, \omega_a)\} f(\omega) d\omega \\ &\quad - (1 - F(\omega_b)) \{y^*(0, 0, \omega_b) - y^*(0, 0, \omega_a)\} \Gamma_0(\omega_b) , \end{aligned}$$

where, by equation (20)

$$\Delta_\tau^R(0, \omega_a, \omega_b) = \frac{1}{1 - I_0} \mathcal{R}(\omega_a, \omega_b) .$$

Hence,

$$\begin{aligned}\Delta_{\tau}^W(0, \omega_a, \omega_b | g) &= \frac{\gamma_0}{1-I_0} \mathcal{R}(\omega_a, \omega_b) \\ &\quad - \int_{\omega_a}^{\omega_b} g(\omega) \tilde{u}_c^0(\cdot) \{y^*(0, 0, \omega) - y^*(0, 0, \omega_a)\} f(\omega) d\omega \\ &\quad - (1 - F(\omega_b)) \{y^*(0, 0, \omega_b) - y^*(0, 0, \omega_a)\} \Gamma_0(\omega_b) ,\end{aligned}$$

We now investigate under which conditions a marginal tax increase of τ over a small interval of types increases welfare. Note that

$$\Delta_{\tau}^W(0, \omega_a, \omega_a | g) = 0 .$$

If $\Delta_{\tau\omega_b}^W(0, \omega_a, \omega_a | g) > 0$ then $\Delta_{\tau}^W(0, \omega_a, \omega_b | g)$ turns positive, if starting from $\omega_a = \omega_b$, we marginally increase ω_b .

Straightforward computations yield

$$\Delta_{\tau\omega_b}^W(0, \omega_a, \omega_a | g) = \frac{\gamma_0}{1-I_0} \mathcal{R}_{\omega_b}(\omega_a, \omega_a) - (1 - F(\omega_a)) \Gamma_0(\omega_a) y_{\omega}^*(0, 0, \omega_a).$$

Recall that

$$\mathcal{R}_{\omega_b}(\omega_a, \omega_a) = T_0'(y^*(0, 0, \omega_a)) y_{\tau}^*(0, 0, \omega_a) f(\omega_a) + (1 - \tilde{I}_0(\omega_a)) y_{\omega}^*(0, 0, \omega_a) (1 - F(\omega_a)) ,$$

so that we can write

$$\Delta_{\tau\omega_b}^W(0, \omega_a, \omega_a | g) = \frac{\gamma_0}{1-I_0} \{T_0'(y^*(0, 0, \omega_a)) y_{\tau}^*(0, 0, \omega_a) f(\omega_a) + (1 - F(\omega_a)) \Phi_0(\omega_a) y_{\omega}^*(0, 0, \omega_a)\} ,$$

where

$$\Phi_0(\omega_a) = 1 - \tilde{I}_0(\omega_a) - (1 - I_0) \frac{\Gamma_0(\omega_a)}{\gamma_0} .$$

Using this expression and the fact that $y_{\tau}^*(0, 0, \omega_0) < 0$ for $\omega_0 \in (\omega_a, \omega_b)$ if ω_b is close to ω_a we obtain the characterization of welfare-increasing reforms in the first statement of Proposition 7.

Proof of Proposition 9

We first show that a small reform is strictly supported by a majority of the population if it is strictly preferred by the median voter. Suppose that $\frac{d\tau_s}{d\tau} > s^0((\omega, \beta)^{0M})$. This also implies $\frac{d\tau_s}{d\tau} > s^0(\omega, \beta)$ for all individuals with $s^0((\omega, \beta)^{0M}) \geq s^0(\omega, \beta)$. By the definition of the status quo median voter $(\omega, \beta)^{0M}$ the mass of taxpayers with this property is equal to $\frac{1}{2}$. Hence, the reform is supported by a majority of the population.

Second, we show that the status quo is weakly preferred by a majority of individuals if it is weakly preferred by the status quo median voter. Suppose that the status quo is weakly preferred by the median voter so that $\frac{d\tau_s}{d\tau} \leq s^0((\omega, \beta)^{0M})$. This also implies $\frac{d\tau_s}{d\tau} \leq s^0(\omega, \beta)$, for all types (ω, β) so that $s^0((\omega, \beta)^{0M}) \leq s^0(\omega, \beta)$. By the definition of $(\omega, \beta)^{0M}$ the mass of taxpayers with this property is equal to $\frac{1}{2}$. Hence, the status quo is weakly preferred by a majority of individuals.

Proof of Proposition 10

We focus without loss of generality on tax increases, i.e. $\tau > 0$ and on a non-decreasing function h .

We first show that a small reform is strictly supported by a majority of the population if it is strictly preferred by the median voter. Suppose that $\Delta_\tau^V((\omega, \theta)^{0M} | 0, h) > 0$. Since $\tilde{u}_c^0(\cdot) > 0$, this implies

$$\Delta_\tau^R(0, h) - h(y^{0M}) > 0 .$$

Since h is a non-decreasing function, this also implies

$$\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega, \theta)) > 0 ,$$

for all (ω, θ) so that $\tilde{y}^0(\omega, \theta) \leq y^{0M}$. By definition of the status quo median voter the mass of taxpayers with $\tilde{y}^0(\omega, \theta) \leq y^{0M}$ is equal to $\frac{1}{2}$. Hence, the reform is supported by a majority of the population.

Second, we show that the status quo is weakly preferred by a majority of individuals if it is weakly preferred by the status quo median voter. Suppose that the status quo is weakly preferred by the median voter so that

$$\Delta_\tau^R(0, h) - h(y^{0M}) \leq 0 .$$

Since h is a non-decreasing function, this also implies

$$\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega, \theta)) \leq 0 ,$$

for all (ω, θ) so that $\tilde{y}^0(\omega, \theta) \leq y^{0M}$. By definition of the status quo median voter the mass of taxpayers with $\tilde{y}^0(\omega, \theta) \leq y^{0M}$ is equal to $\frac{1}{2}$. Hence, the status quo is weakly preferred by a majority of individuals.

C Bunching and non-negativity constraints

C.1 Bunching

Proposition C.1 below extends Proposition 4 in the body of the text so as to allow for bunching in the characterization of revenue-increasing reforms. We also provide a formal proof of Proposition C.1. We leave the extensions of Propositions 5, 6 and 7 to the reader. These extensions simply require to replace the function $\tilde{\omega}^0$ by its analog $\tilde{\omega}^{0b} : y \mapsto \max\{\omega \mid y^*(0, 0, \omega) = y\}$ that takes account of the possibility of bunching.

Proposition C.1 *Define $\mathcal{D}^{Rb}(y) := D^R(\tilde{\omega}^{0b}(y))$.*

1. *Suppose that there is an income level y_0 so that $T_0'(y_0) < \mathcal{D}^{Rb}(y_0)$. Then there exists a revenue-increasing reform with $\tau > 0$, and $y_a < y_0 < y_b$.*
2. *Suppose that there is an income level y_0 so that $T_0'(y_0) > \mathcal{D}^{Rb}(y_0)$. Then there exists a revenue-increasing reform with $\tau < 0$, and $y_a < y_0 < y_b$.*

Proof. We prove only the first statement in the Proposition. The proof of the second statement is analogous.

We consider a reform in the (τ, y_a, y_b) -class with $\tau > 0$ and assume that, prior to the reform, there is a set of types $[\underline{\omega}_a(0), \bar{\omega}_a(0)]$ who bunch at y_a . More formally, for all $\omega \in [\underline{\omega}_a(0), \bar{\omega}_a(0)]$, $y^*(0, 0, \omega) = \tilde{y}^0(\omega) = y_a$. We also assume, without loss of generality, that there is no further bunching between y_a and y_b so that the function \tilde{y}^0 is strictly increasing for $\omega \in (\bar{\omega}_a(0), \omega_b(0)]$. The reform will affect the set of types who bunch at y_a and we denote by $\underline{\omega}_a(\tau)$ and $\bar{\omega}_a(\tau)$ the minimal and the maximal type, respectively, who chose an earnings level of y_a after the reform.

We first look at the implication of such a reform for tax revenue. Again, we decompose the change in tax revenue.

$$\Delta^R(\tau, y_a, y_b) = \Delta^{R1}(\tau, y_a, y_b) + \Delta^{R2}(\tau, y_a, y_b) + \Delta^{R3}(\tau, y_a, y_b) + \Delta^{R4}(\tau, y_a, y_b) ,$$

where

$$\Delta^{R1}(\tau, y_a, y_b) = \int_{\underline{\omega}}^{\underline{\omega}_a(\tau)} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega ,$$

$$\Delta^{R2}(\tau, y_a, y_b) = \int_{\underline{\omega}_a(\tau)}^{\bar{\omega}_a(\tau)} \{T_1(y_a) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega ,$$

$$\Delta^{R3}(\tau, y_a, y_b) = \int_{\bar{\omega}_a(\tau)}^{\omega_b(\tau)} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega ,$$

and

$$\Delta^{R4}(\tau, y_a, y_b) = \int_{\omega_b(\tau)}^{\bar{\omega}} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega .$$

Following the same steps as in the proof of Proposition 4, we obtain a characterization of $\Delta_\tau^R(0, y_a, y_b)$,

$$\Delta_\tau^R(0, y_a, y_b) = \frac{1}{1 - I_0^b(y_a, y_b)} \mathcal{R}^b(y_a, y_b) ,$$

where

$$I_0^b(y_a, y_b) = \int_{\underline{\omega}}^{\omega_a(0)} T_0'(\cdot) y_e^*(0, 0, \omega) f(\omega) d\omega + \int_{\bar{\omega}_a(0)}^{\bar{\omega}} T_0'(\cdot) y_e^*(0, 0, \omega) f(\omega) d\omega ,$$

and

$$\begin{aligned} \mathcal{R}^b(y_a, y_b) &= \int_{\bar{\omega}_a(0)}^{\omega_b(0)} T_0'(\cdot) y_\tau^*(0, 0, \omega) f(\omega) d\omega + \int_{\bar{\omega}_a(0)}^{\omega_b(0)} \{y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\ &\quad + (y_b - y_a)(1 - F(\omega_b(0))) - (y_b - y_a) \int_{\omega_b(0)}^{\bar{\omega}} T_0'(\cdot) y_e^*(0, 0, \omega) f(\omega) d\omega . \end{aligned}$$

The superscripts b indicate that these expressions are the analogs to I_0 and \mathcal{R} in the proof of Proposition 4 that take account of bunching.

In the following to save on notation, we write $\underline{\omega}_a$, $\bar{\omega}_a$ and ω_b rather than $\underline{\omega}_a(0)$, $\bar{\omega}_a(0)$ and $\omega_b(0)$. Again, we perform a change in variables and interpret $I_0^b(y_a, y_b)$ as a function of the parameters $\underline{\omega}_a$ and $\bar{\omega}_a$ and $\mathcal{R}^b(y_a, y_b)$ as a function of $\bar{\omega}_a$ and ω_b and write, with some abuse of notation,

$$I_0^b(\underline{\omega}_a, \bar{\omega}_a) = \int_{\underline{\omega}}^{\underline{\omega}_a} T_0'(\cdot) y_e^*(0, 0, \omega) f(\omega) d\omega + \int_{\bar{\omega}_a}^{\bar{\omega}} T_0'(\cdot) y_e^*(0, 0, \omega) f(\omega) d\omega ,$$

and

$$\begin{aligned}\mathcal{R}^b(\bar{\omega}_a, \omega_b) &= \int_{\bar{\omega}_a}^{\omega_b} T'_0(\cdot) y_\tau^*(0, 0, \omega) f(\omega) d\omega + \int_{\bar{\omega}_a}^{\omega_b} \{y^*(0, 0, \omega) - y^*(0, 0, \bar{\omega}_a)\} f(\omega) d\omega \\ &\quad + (y^*(0, 0, \omega_b) - y^*(0, 0, \bar{\omega}_a))(1 - F(\omega_b)) \\ &\quad - (y^*(0, 0, \omega_b) - y^*(0, 0, \bar{\omega}_a)) \int_{\bar{\omega}_b}^{\bar{\omega}} T'_0(\cdot) y_e^*(0, 0, \omega) f(\omega) d\omega .\end{aligned}$$

Hence,

$$\Delta_\tau^R(0, \underline{\omega}_a, \bar{\omega}_a, \omega_b) = \frac{1}{1 - I_0^b(\underline{\omega}_a, \bar{\omega}_a)} \mathcal{R}^b(\bar{\omega}_a, \omega_b) ,$$

Moreover, note that since there is, by assumption, no further bunching between y_a and y_b ,

$$\mathcal{R}^b(\bar{\omega}_a, \omega_b) = \mathcal{R}(\bar{\omega}_a, \omega_b) .$$

We now investigate the conditions under which

$$\Delta_{\tau\omega_b}^R(0, \underline{\omega}_a, \bar{\omega}_a, \omega_a) > 0 .$$

If this inequality holds then there exists a reform (τ, y_a, y_b) for some $y_b > y_a$ that leads to an increase of tax revenue. Also note that $\Delta_{\tau\omega_b}^R(0, \underline{\omega}_a, \bar{\omega}_a, \omega_a) > 0$ holds if and only if

$$\mathcal{R}_{\omega_b}(\bar{\omega}_a, \bar{\omega}_a) > 0 .$$

The condition under which this last inequality holds have been characterized in the proof of Proposition 4.

If

$$T'_0(y_a) < -\frac{1 - F(\bar{\omega}_a)}{f(\bar{\omega}_a)} \left(1 - \tilde{I}_0(\bar{\omega}_a)\right) \frac{y_\omega^*(0, 0, \bar{\omega}_a)}{y_\tau^*(0, 0, \bar{\omega}_a)} ,$$

or, equivalently,

$$T'_0(y_a) < D^R(\bar{\omega}_a)$$

Using the function $\tilde{\omega}^{0b} : y \mapsto \max \omega \mid y^*(0, 0, \omega) = y$ we can express this also as

$$T'_0(y_a) < D^R(\tilde{\omega}^{0b}(y_a)) ,$$

or as

$$T'_0(y_a) < D^R(\tilde{\omega}^{0b}(y_a)) ,$$

which proves statement 1. in Proposition C.1.

C.2 Non-negativity constraints

Binding non-negativity constraints on earnings are a particular type of bunching. The behavioral responses to a reform in the (τ, y_a, y_b) -class with $y_a > 0$ and $\tau > 0$ then look as follows: Individuals with $\omega \leq \hat{\omega}^P(\tau)$ choose earnings of zero after the reform, individuals with $\omega \in (\hat{\omega}^P(\tau), \omega_a(\tau))$ choose $y \in (0, y_a)$ after the reform, individuals with $\omega \in [\omega_a(\tau), \bar{\omega}_a(\tau)]$ choose $y = y_a$ after the reform, individuals with $\omega \in (\bar{\omega}_a(\tau), \omega_b(\tau))$ choose $y \in (y_a, y_b)$, and individuals with $\omega \geq \omega_b(\tau)$ choose $y \geq y_b$.

We leave it to the reader to verify that a small reform in the (τ, y_a, y_b) -class raises revenue if and only if the conditions in Proposition 4 are fulfilled. While the accounting of behavioral responses has to take account of individuals with no income, the analysis in the end boils down to an analysis of the conditions under which $\mathcal{R}_{\omega_b}(\omega_a, \omega_a) > 0$ holds, just as in the proof of Proposition 4. If we consider instead a reform in the (τ, y_a, y_b) -class with $y_a = 0$, we modify marginal tax rates at a point of bunching. In this case the conditions in Proposition C.1 evaluated for $y_a = 0$ clarify whether such a reform raises tax revenue.

Once the revenue implications are clear, extensions of Propositions 5, 6 and 7 that allow for binding non-negativity constraints can be obtained along the same lines as in the body of the text.

D Additional details for the proof of Proposition 4

We provide the details for one part of the Proof of Proposition 4 we omitted in Appendix B.

We provide a characterization of $\Delta_\tau^R(\tau, y_a, y_b)$ by looking at the different subsets of the population separately. We then evaluate $\Delta_\tau^R(\tau, y_a, y_b)$ at $\tau = 0$ since $\Delta_\tau^R(0, y_a, y_b)$ gives the increase in tax revenue if we start to increase marginal tax rates for incomes in the interval $[y_a, y_b]$. Since

$$\Delta_\tau^R(\tau, y_a, y_b) = \Delta_\tau^{R1}(\tau, y_a, y_b) + \Delta_\tau^{R2}(\tau, y_a, y_b) + \Delta_\tau^{R3}(\tau, y_a, y_b) + \Delta_\tau^{R4}(\tau, y_a, y_b) ,$$

we can characterize $\Delta_\tau^R(\tau, y_a, y_b)$ by looking at each subset of types separately

For the first subset:

$$\begin{aligned} \Delta_\tau^{R1}(\tau, y_a, y_b) &= \Delta_\tau^R(\cdot) \int_{\underline{\omega}}^{\omega_a(\tau)} T_0'(y^*(\Delta^R(\cdot), 0, \omega)) y_e^*(\Delta^R(\cdot), 0, \omega) f(\omega) d\omega \\ &\quad + \{T_0(y_a) - T_0(y^*(0, 0, \omega_a(\tau)))\} f(\omega_a(\tau)) \omega_a'(\tau) , \end{aligned}$$

where $\omega_a'(\tau)$ is the derivative of $\omega_a(\tau)$. Analogously we derive

$$\begin{aligned} \Delta_\tau^{R2}(\tau, y_a, y_b) &= \{T_0(y_a) - T_0(y^*(0, 0, \hat{\omega}_a(\tau)))\} f(\hat{\omega}_a(\tau)) \hat{\omega}_a'(\tau) \\ &\quad - \{T_0(y_a) - T_0(y^*(0, 0, \omega_a(\tau)))\} f(\omega_a(\tau)) \omega_a'(\tau) , \end{aligned}$$

$$\begin{aligned}
\Delta_\tau^{R3}(\tau, y_a, y_b) &= \int_{\hat{\omega}_a(\tau)}^{\omega_b(\tau)} \{T'_0(y^*(\Delta^R(\cdot), \tau, \omega)) + \tau\} \{y_e^*(\Delta^R(\cdot), \tau, \omega) \Delta_\tau^R(\cdot) + y_\tau^*(\Delta^R(\cdot), \tau, \omega)\} f(\omega) d\omega \\
&\quad + \int_{\hat{\omega}_a(\tau)}^{\omega_b(\tau)} \{y^*(\Delta^R(\cdot), \tau, \omega) - y_a\} f(\omega) d\omega \\
&\quad - \{T_0(y_a) - T_0(y^*(0, 0, \hat{\omega}_a(\tau)))\} f(\hat{\omega}_a(\tau)) \hat{\omega}'_a(\tau) \\
&\quad + \{T_0(y_b) + \tau(y_b - y_a) - T_0(y^*(0, 0, \omega_b(\tau)))\} f(\omega_b(\tau)) \omega'_b(\tau) \\
&= \Delta_\tau^R(\cdot) \int_{\hat{\omega}_a(\tau)}^{\omega_b(\tau)} T'_0(y^*(\Delta^R(\cdot), \tau, \omega)) y_e^*(\Delta^R(\cdot), \tau, \omega) f(\omega) d\omega \\
&\quad + \int_{\hat{\omega}_a(\tau)}^{\omega_b(\tau)} T'_0(y^*(\Delta^R(\cdot), \tau, \omega)) y_\tau^*(\Delta^R(\cdot), \tau, \omega) f(\omega) d\omega \\
&\quad + \tau \int_{\hat{\omega}_a(\tau)}^{\omega_b(\tau)} \{y_e^*(\Delta^R(\cdot), \tau, \omega) \Delta_\tau^R(\cdot) + y_\tau^*(\Delta^R(\cdot), \tau, \omega)\} f(\omega) d\omega \\
&\quad + \int_{\hat{\omega}_a(\tau)}^{\omega_b(\tau)} \{y^*(\Delta^R(\cdot), \tau, \omega) - y_a\} f(\omega) d\omega \\
&\quad - \{T_0(y_a) - T_0(y^*(0, 0, \hat{\omega}_a(\tau)))\} f(\hat{\omega}_a(\tau)) \hat{\omega}'_a(\tau) \\
&\quad + \{T_0(y_b) + \tau(y_b - y_a) - T_0(y^*(0, 0, \omega_b(\tau)))\} f(\omega_b(\tau)) \omega'_b(\tau),
\end{aligned}$$

and

$$\begin{aligned}
\Delta_\tau^{R4}(\tau, y_a, y_b) &= (y_b - y_a)(1 - F(w_b(\tau))) \\
&\quad \Delta_\tau^R(\cdot) \int_{\omega_b(\tau)}^{\bar{\omega}} T'_0((\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega)) y_e^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega) f(\omega) d\omega \\
&\quad - (y_b - y_a) \int_{\omega_b(\tau)}^{\bar{\omega}} T'_0((\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega)) y_e^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega) f(\omega) d\omega \\
&\quad - \{T_0(y_b) + \tau(y_b - y_a) - T_0(y^*(0, 0, \omega_b(\tau)))\} f(\omega_b(\tau)) \omega'_b(\tau).
\end{aligned}$$

Define

$$\begin{aligned}
I(\tau, y_a, y_b) &:= \int_{\underline{\omega}}^{\omega_a(\tau)} T'_0(y^*(\Delta^R(\cdot), 0, \omega)) y_e^*(\Delta^R(\cdot), 0, \omega) f(\omega) d\omega \\
&\quad + \int_{\hat{\omega}_a(\tau)}^{\omega_b(\tau)} T'_0(y^*(\Delta^R(\cdot), \tau, \omega)) y_e^*(\Delta^R(\cdot), \tau, \omega) f(\omega) d\omega \\
&\quad + \int_{\omega_b(\tau)}^{\bar{\omega}} T'_0((\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega)) y_e^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega) f(\omega) d\omega.
\end{aligned}$$

Upon collecting terms we find

$$\begin{aligned}
\Delta_\tau^R(\tau, y_a, y_b) &= \Delta_\tau^R(\tau, y_a, y_b) I(\tau, y_a, y_b) \\
&\quad + \int_{\hat{\omega}_a(\tau)}^{\omega_b(\tau)} T'_0(y^*(\Delta^R(\cdot), \tau, \omega)) y_\tau^*(\Delta^R(\cdot), \tau, \omega) f(\omega) d\omega \\
&\quad + \tau \int_{\hat{\omega}_a(\tau)}^{\omega_b(\tau)} \{y_e^*(\Delta^R(\cdot), \tau, \omega) \Delta_\tau^R(\cdot) + y_\tau^*(\Delta^R(\cdot), \tau, \omega)\} f(\omega) d\omega \\
&\quad + \int_{\hat{\omega}_a(\tau)}^{\omega_b(\tau)} \{y^*(\Delta^R(\cdot), \tau, \omega) - y_a\} f(\omega) d\omega \\
&\quad + (y_b - y_a)(1 - F(w_b(\tau))) \\
&\quad - (y_b - y_a) \int_{\omega_b(\tau)}^{\bar{\omega}} T'_0(y^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega)) y_e^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega) f(\omega) d\omega.
\end{aligned}$$

If we evaluate this expression at $\tau = 0$ and assume that there is no bunching at y_a under the initial tax schedule T_0 so that $\omega_a(0) = \hat{\omega}_a(0)$ we obtain

$$\begin{aligned}
\Delta_\tau^R(0, y_a, y_b) &= \Delta_\tau^R(0, y_a, y_b) I(0, y_a, y_b) \\
&\quad + \int_{\omega_a(0)}^{\omega_b(0)} T'_0(y^*(0, 0, \omega)) y_\tau^*(0, 0, \omega) f(\omega) d\omega \\
&\quad + \int_{\omega_a(0)}^{\omega_b(0)} \{y^*(0, 0, \omega) - y_a\} f(\omega) d\omega \\
&\quad + (y_b - y_a)(1 - F(w_b(0))) \\
&\quad - (y_b - y_a) \int_{\omega_b(0)}^{\bar{\omega}} T'_0(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega,
\end{aligned}$$

where

$$I_0 = \tilde{I}(\underline{\omega}) = I(0, y_a, y_b) = \int_{\underline{\omega}}^{\bar{\omega}} T'_0(y^*(0, 0, \omega)) y_e^*(0, 0, \omega) f(\omega) d\omega$$

is an expression that no longer depends on the parameters y_a and y_b , thus, we use a shorter notation and simply write I_0 . The rest of the proof is in Appendix B.